



FURTHER MATHEMATICS

9795/02

Paper 2 Further Applications of Mathematics

May/June 2013

3 hours

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF20)



READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use 10 m s^{-2} .

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 120.

This document consists of **4** printed pages.



Section A: Probability (60 marks)

- 1** A company hires out narrowboats on a canal. It may be assumed that demands to hire a narrowboat occur independently and randomly at a constant mean rate of 25 per week. Using a suitable normal approximation, find
- (i) the probability that 15 or fewer narrowboats are hired out during a certain week, [4]
 - (ii) the number of narrowboats that the company needs to have available for a week in order that the probability of running out of boats is 0.05 or less. [3]
- 2**
- (i) The heights of boys in Year 9 are normally distributed with mean 156 cm and standard deviation 8 cm. The heights of girls in Year 10 are, independently, normally distributed with mean 160 cm and standard deviation 7 cm. Find the probability that the mean height of a random sample of 9 boys in Year 9 exceeds the mean height of a random sample of 16 girls in Year 10. [8]
 - (ii) State why the distributions of the sample means are normally distributed. [1]
- 3**
- (i) Given that $X \sim \text{Po}(5)$, find $P(X > 6 \mid X > 3)$. [3]
 - (ii) Given that $Y \sim \text{Po}(\lambda)$ and $P(Y \leq 1) = \frac{1}{2}$, show that λ satisfies the equation $\lambda = \ln\{2(1 + \lambda)\}$. [3]
 - (iii) Starting with a suitable approximation from the table of cumulative Poisson probabilities, use iteration to find λ correct to 3 decimal places. [3]
- 4** The broadband speed in village P was measured on 8 randomly selected occasions and the broadband speed in village Q was measured on 6 randomly selected occasions. The results, measured in megabits per second, are shown below.
- | | | | | | | | | |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Village P : | 4.8 | 3.5 | 2.9 | 3.7 | 4.2 | 4.6 | 5.1 | 3.3 |
| Village Q : | 2.4 | 1.9 | 2.3 | 3.1 | 2.7 | 2.9 | | |
- (i) Calculate a 90% confidence interval for the difference in mean broadband speed in these two villages. [8]
 - (ii) State two assumptions that you have made in carrying out the calculation. [2]
- 5** The discrete random variable X has probability generating function given by
- $$G_X(t) = k(5t^{-1} + 3 + 2t^2),$$
- where k is a constant.
- (i) Find
 - (a) the value of k , [1]
 - (b) the modal value of X . [1]
 - (ii) The random variables X_1 and X_2 are independent observations of X .
 - (a) Write down the probability generating function of Y , where $Y = X_1 + X_2$. [1]
 - (b) Use your answer to part (ii)(a) to find $E(Y)$ and $\text{Var}(Y)$. [8]

- 6 A rectangle of area $Y \text{ m}^2$ has a perimeter of 16 m and a side of length $X \text{ m}$, where X is a random variable with probability density function, f , given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

- (i) Obtain the cumulative distribution function, F , of X . [2]

- (ii) Show that

$$16 - Y = (4 - X)^2,$$

and deduce that the probability density function of the random variable Y is

$$g(y) = \begin{cases} \frac{1}{4\sqrt{16-y}} & 0 \leq y \leq 12, \\ 0 & \text{otherwise.} \end{cases} \quad [5]$$

- (iii) Find the median of Y . [2]

- (iv) Find $E(Y)$. [5]

Section B: Mechanics (60 marks)

- 7 Find the power required to pump 3 m^3 of water per minute from a depth of 25 m and deliver it through a circular pipe of diameter 10 cm. Assume that friction may be neglected and that the density of water is 1000 kg m^{-3} . [8]

- 8 A particle, P , is moving in a straight line with simple harmonic motion about a centre O . When P is at the point A , 2 m from O , it has speed $4\sqrt{5} \text{ m s}^{-1}$. When P is at the point B , $\sqrt{5} \text{ m}$ from O , it has speed 8 m s^{-1} .

- (i) Find the amplitude and period of the motion. [6]

- (ii) Given that A and B are on opposite sides of O , find the time taken for P to travel directly from A to B . [3]

- 9 A particle of mass 2 kg is moving along the x -axis, which is horizontal, against a resistive force which is proportional to the cube of the speed of the particle at any instant. At time t seconds the particle's velocity is $v \text{ m s}^{-1}$ and its displacement is $x \text{ m}$. When $t = 0$, $x = 0$, $v = 4$ and the retardation is 2 m s^{-2} .

- (i) Show that

$$\frac{1}{v} = \frac{x + 8}{32}. \quad [6]$$

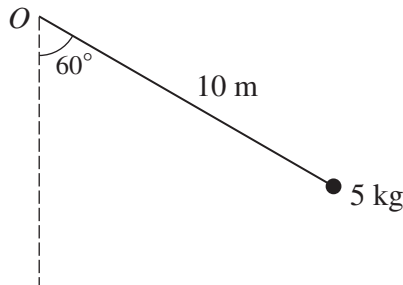
- (ii) Find the time taken to cover the first 8 metres. [4]

[Questions 10, 11 and 12 are printed on the next page.]

10 Ship A is 15 km due south of ship B . Ship B is travelling at 20 km h^{-1} on a bearing of 300° . Ship A is travelling at 16 km h^{-1} . Find

- (i) the bearing, to the nearest degree, that A must take in order to get as close as possible to B , [4]
 (ii) the time, in minutes, that it takes for the ships to be as close as possible. [6]

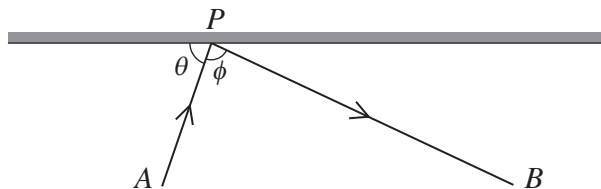
11



One end of a light inextensible string of length 10 m is attached to a fixed point O . A particle of mass 5 kg is attached to the other end of the string. The particle rests in equilibrium below O . The particle is pulled aside until the string makes an angle of 60° with the downward vertical and released from rest (see diagram). At the instant when the string makes an angle $\cos^{-1}\left(\frac{4}{5}\right)$ with the downward vertical,

- (i) find the speed of the particle and the tension in the string, [7]
 (ii) show that the magnitude of the acceleration of the particle is $6\sqrt{2} \text{ m s}^{-2}$. [4]

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A small smooth sphere is projected from a point A across a smooth horizontal surface. The sphere strikes a smooth vertical wall at the point P . The acute angle between the direction of motion of the sphere and the wall is θ . After the impact, the sphere passes through the point B , where angle $APB = \phi$ (see diagram). The coefficient of restitution between the sphere and the wall is e .

- (i) Given that $\theta = \tan^{-1} 3$ and $\phi = 90^\circ$, find the exact value of e . [6]
 (ii) Given instead that $e = \frac{2}{3}$ and $\phi = 45^\circ$, show that $\theta = \tan^{-1} 3$. [6]