



FURTHER MATHEMATICS

9795/01

Paper 1 Further Pure Mathematics

May/June 2016

MARK SCHEME

Maximum Mark: 120

Published

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Question	Answer	Marks	Notes
1	$\sum_{r=1}^n (8r^3 + r) \equiv 8 \sum_{r=1}^n r^3 + \sum_{r=1}^n r$ $\equiv 8 \times \frac{1}{4} n^2 (n+1)^2 + \frac{1}{2} n(n+1)$ $\equiv \frac{1}{2} n(n+1) \{4n^2 + 4n + 1\}$ $\equiv \frac{1}{2} n(n+1)(2n+1)^2$	M1 M1 M1 A1 [4]	Splitting into separate series Both used good factorisation attempt Legitimate (AG)
2	$\begin{pmatrix} 6 \\ 2 \\ 5 \end{pmatrix} \times \begin{pmatrix} -6 \\ 1 \\ 4 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ -18 \\ 6 \end{pmatrix}$ <p>Shortest Distance = $\mathbf{(b-a)} \cdot \hat{\mathbf{n}}$</p> $= \frac{1}{19} \begin{pmatrix} 10 \\ -2 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -18 \\ 6 \end{pmatrix} = \frac{1}{19} (10 + 36 + 30)$ $= 4$ <p>Alternative method:</p> <p>M1 A1 for common normal $\mathbf{i} - 18\mathbf{j} + 6\mathbf{k}$ M1 A1 for parallel planes $x - 18y + 6z = -55$ and -131</p> <p>M1 A1 for Sh.D formula, $\frac{ 131 - 55 }{ \mathbf{n} } = \frac{76}{19} = 4$</p>	M1 A1 M1 B1 B1 A1 [6]	Attempt at vector products of the d.v.s (any suitable multiple) $ \hat{\mathbf{n}} $ correct Sc. Prod. ft correct
3 (i)	$\frac{2x^2 - x - 1}{2x - 3} = k \Rightarrow 2x^2 - (2k+1)x + (3k-1) = 0$ <p>For non-real x, $(2k+1)^2 - 8(3k-1) < 0$</p> $4k^2 - 20k + 9 < 0 \Rightarrow (2k-1)(2k-9) < 0$ $\Rightarrow \text{no curve for } \frac{1}{2} < k = y < \frac{9}{2}$	B1 M1 M1 A1 [4]	(AG) Shown legitimately Considering discriminant (or equivalent) Solving from $\Delta < 0$ (AG) Must be satisfactorily explained
(ii)	<p>TPs at $y = \frac{1}{2}$ $y = \frac{9}{2}$</p> <p>i.e. $2x^2 - 2x + \frac{1}{2} = 0$ $2x^2 - 10x + \frac{25}{2} = 0$</p> $x = \frac{1}{2}$ $x = \frac{5}{2}$ <p>Alternative method:</p> <p>when $\Delta = 0$, M1 $x = -\frac{b}{2a} = \frac{2k+1}{4}$</p> <p>M1 $\Rightarrow x = \frac{1}{2}$ ($y = \frac{1}{2}$) & $x = \frac{5}{2}$ ($y = \frac{9}{2}$) A1 A1</p> <p>Note: For finding TP's via $\frac{dy}{dx} = 0$, max. M1 A1 since qn. asks for a "deduce" method</p>	M1 M1 A1A1 [4]	First y (k) substituted back Second y (k) substituted back

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Question	Answer	Marks	Notes
4 (i)	Attempt at det(M) Det = 0 <i>Shown</i>	M1 A1 [2]	(Or via full alternative algebraic method)
(ii)	$-x + 3y + z = 1$ $5x - y + 2z = 16$ $-x + y = -2$ parametrisation attempt (or equivalent) started: e.g. set $x = \lambda$, then $y = \lambda - 2$ complete attempt: $z = 1 + \lambda - 3\lambda + 6 = 7 - 2\lambda$ all correct (p.v. and d.v.) ... may be in vector line eqn. form: $\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ <u>Alternative method 1:</u> B1 as above, followed by (e.g.): Finding two distinct points on the solution line; e.g. (2, 0, 3), (0, -2, 7) M1 A1 Then eqn. of line containing these 2 points M1 A1 possibly ft for line (of intersection) of 3 planes (given by the 3 eqns.) B1 <u>Alternative method 2:</u> B1 as above, followed by: Vector product of any two plane normals M1A1 Finding coords. or p.v. of any pt. on line B1 Eqn. of line using these results appropriately B1 for line (of intersection) of 3 planes (given by the 3 eqns.) B1	B1 M1 M1 A1A1 [6]	for all three
5	Aux. Eqn. $m^2 - 4m + 5 = 0$ $m = 2 \pm i$ Comp. Fn. is $y_C = e^{2x} (A \cos x + B \sin x)$ For Part. Intgl. try $y = y_p = a e^{2x}$ Both $y' = 2a e^{2x}$ and $y'' = 4a e^{2x}$ Subst ^g . into given d.e. & solving to find a : $y_p = 24e^{2x}$ Gen. Soln. $y = e^{2x} (A \cos x + B \sin x + 24)$	M1 A1 B1ft B1 B1 M1 A1 B1ft [8]	Including solving attempt $(4a - 8a + 5a) e^{2x} = 24e^{2x}$ $y_C + y_P$ provided y_C has 2 arbitrary constants and y_P has none. Also, A, B must be real here


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6 (i)	For $f(x) = \sinh x + \sin x - 3x$, $f(2.5) = -0.851... < 0$ and $f(3) = 1.159... > 0$ Change-of-sign (for a continuous fn.) $\Rightarrow 2.5 < \alpha < 3$	M1 A1 [2]	or LHS < RHS and then LHS > RHS All correctly shown/explained
	(ii) $\sinh x + \sin x = \left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \dots \right) + \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots \right)$ $= 2x + \frac{x^5}{60} + \dots$ $2x + \frac{x^5}{60} = 3x \Rightarrow (x \neq 0) x^4 = 60$ $\Rightarrow \alpha \approx \sqrt[4]{60} \quad (2.783 \ 158 \dots)$	M1 A1 B1 [3]	for use of both series (attempted) (AG) shown legitimately
	(iii) Using $2x + \frac{x^5}{60} + \frac{x^9}{181 \ 440} = 3x$ with $x \neq 0$ Solving as a quadratic in x^4 $\alpha \approx 2.769 \ 8$ (to 4 d.p.) [c.f. actual root 2.769 7 to 4 d.p.]	M1 M1 A1 [3]	$x^8 + 3024x^4 - 181 \ 440 = 0$ from $x^4 = \sqrt{2 \ 467 \ 584} - 1512$, $x = \sqrt[4]{58.854 \ 5...}$
7 (i)	$ z^3 = 2\sqrt{2}$ $\arg(z^3) = \frac{1}{4}\pi$ $\Rightarrow z = \left(\sqrt{2}, \frac{1}{12}\pi\right)$ cube-rooting modulus; $\arg \div 3$ Other two roots: $\left(\sqrt{2}, \frac{3}{4}\pi\right)$ and $\left(\sqrt{2}, \frac{17}{12}\pi\right)$	B1B1 M1M1 A1A1 [6]	(in at least the first case)
	(ii) Equilateral Δ with vertices in approx. correct places Area = $3 \times \frac{1}{2} \times \sqrt{2} \times \sqrt{2} \sin\left(\frac{2}{3}\pi\right) = \frac{3}{2}\sqrt{3}$ Accept awrt 2.60 (3 s.f.) from correct working	B1 M1A1 [3]	Give M1 for any correct area

Question	Answer	Marks	Notes																																																	
8 (i) (a)	<table border="1"> <tr><td><i>G</i></td><td>1</td><td>2</td><td>4</td><td>8</td><td>16</td><td>32</td></tr> <tr><td>1</td><td>1</td><td>2</td><td>4</td><td>8</td><td>16</td><td>32</td></tr> <tr><td>2</td><td>2</td><td>4</td><td>8</td><td>16</td><td>32</td><td>1</td></tr> <tr><td>4</td><td>4</td><td>8</td><td>16</td><td>32</td><td>1</td><td>2</td></tr> <tr><td>8</td><td>8</td><td>16</td><td>32</td><td>1</td><td>2</td><td>4</td></tr> <tr><td>16</td><td>16</td><td>32</td><td>1</td><td>2</td><td>4</td><td>8</td></tr> <tr><td>32</td><td>32</td><td>1</td><td>2</td><td>4</td><td>8</td><td>16</td></tr> </table>	<i>G</i>	1	2	4	8	16	32	1	1	2	4	8	16	32	2	2	4	8	16	32	1	4	4	8	16	32	1	2	8	8	16	32	1	2	4	16	16	32	1	2	4	8	32	32	1	2	4	8	16		
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			M1	for mostly correct																																																
			A1	for all correct																																																
		[2]																																																		
(b)	<p>(S, \times_{63}) closed, since no new elements in table</p> <p>\times_{63} is associative (given)</p> <p>1 is the identity element</p> <p>Each (non-identity) element has a unique inverse:</p> <p>$2 \leftrightarrow 32, 4 \leftrightarrow 16$ and 8 is self-inverse</p>	B1																																																		
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		B1	All must be identified																																																	
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(ii) (a)	<table border="1"> <tr><td><i>H</i></td><td><i>e</i></td><td><i>x</i></td><td><i>y</i></td><td>y^2</td><td><i>xy</i></td><td><i>yx</i></td></tr> <tr><td><i>e</i></td><td><i>e</i></td><td><i>x</i></td><td><i>y</i></td><td>y^2</td><td><i>xy</i></td><td><i>yx</i></td></tr> <tr><td><i>x</i></td><td><i>x</i></td><td><i>e</i></td><td><i>xy</i></td><td><i>yx</i></td><td><i>y</i></td><td>y^2</td></tr> <tr><td><i>y</i></td><td><i>y</i></td><td><i>yx</i></td><td>y^2</td><td><i>e</i></td><td><i>x</i></td><td><i>xy</i></td></tr> <tr><td>y^2</td><td>y^2</td><td><i>xy</i></td><td><i>e</i></td><td><i>y</i></td><td><i>yx</i></td><td><i>x</i></td></tr> <tr><td><i>xy</i></td><td><i>xy</i></td><td>y^2</td><td><i>yx</i></td><td><i>x</i></td><td><i>e</i></td><td><i>y</i></td></tr> <tr><td><i>yx</i></td><td><i>yx</i></td><td><i>y</i></td><td><i>x</i></td><td><i>xy</i></td><td>y^2</td><td><i>e</i></td></tr> </table>	<i>H</i>	<i>e</i>	<i>x</i>	<i>y</i>	y^2	<i>xy</i>	<i>yx</i>	<i>e</i>	<i>e</i>	<i>x</i>	<i>y</i>	y^2	<i>xy</i>	<i>yx</i>	<i>x</i>	<i>x</i>	<i>e</i>	<i>xy</i>	<i>yx</i>	<i>y</i>	y^2	<i>y</i>	<i>y</i>	<i>yx</i>	y^2	<i>e</i>	<i>x</i>	<i>xy</i>	y^2	y^2	<i>xy</i>	<i>e</i>	<i>y</i>	<i>yx</i>	<i>x</i>	<i>xy</i>	<i>xy</i>	y^2	<i>yx</i>	<i>x</i>	<i>e</i>	<i>y</i>	<i>yx</i>	<i>yx</i>	<i>y</i>	<i>x</i>	<i>xy</i>	y^2	<i>e</i>		
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			B1	for last 3 elements (any forms)																																																
			B1	for identity row/column (green)																																																
		B1	for easy elements (gold) or ≥ 14 others																																																	
		B1	for all																																																	
		[4]																																																		
(b)	<p>Proper subgroups of H are (condone inclusion of $\{e\}$ and H):</p> <p>$\{e, x\}, \{e, xy\}, \{e, yx\}$ and $\{e, y, y^2\}$</p>	B1B1	B1 Any 2; +B1 all 4 and no extras																																																	
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(c)	<p>G and H are NOT isomorphic</p> <p>e.g. Different numbers of self-inverse elements / elements of order 3</p> <p>or G cyclic, H non-cyclic or G abelian, H non-abelian</p>	B1	Correct conclusion WITH a valid reason																																																	
		[1]																																																		

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9 (i)	$\alpha + \beta + \gamma = a$, $\alpha\beta + \beta\gamma + \gamma\alpha = b$ and $\alpha\beta\gamma = c$	B1B1 [2]	B1 any 2 correct; + B1 all 3 correct
(ii)	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ $= a^2 - 2b$ $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$ $= b^2 - 2ac$	M1 A1 M1 A1 [4]	
(iii)	$(\alpha - 2\beta\gamma)(\beta - 2\gamma\alpha)(\gamma - 2\alpha\beta)$ $= (\alpha\beta - 2\beta^2\gamma - 2\alpha^2\gamma + 4\gamma^2\alpha\beta)(\gamma - 2\alpha\beta)$ $= \alpha\beta\gamma - 2(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2) + 4\alpha\beta\gamma(\alpha^2 + \beta^2 + \gamma^2) - 8(\alpha\beta\gamma)^2$ $= c - 2(b^2 - 2ac) + 4c(a^2 - 2b) - 8c^2$ $= c(1 + 4a + 4a^2) - 2(b^2 + 4bc + 4c^2)$ $= c(2a + 1)^2 - 2(b + 2c)^2$ <p><u>Alternative method:</u></p> <p>Using $\alpha\beta\gamma = c$,</p> $(\alpha - 2\beta\gamma)(\beta - 2\gamma\alpha)(\gamma - 2\alpha\beta)$ $= \left(\alpha - \frac{2c}{\alpha}\right)\left(\beta - \frac{2c}{\beta}\right)\left(\gamma - \frac{2c}{\gamma}\right)$ $= \frac{1}{\alpha\beta\gamma}(\alpha^2 - 2c)(\beta^2 - 2c)(\gamma^2 - 2c) =$ $\frac{1}{c}((\alpha\beta\gamma)^2 - 2c\sum\alpha^2\beta^2 + 4c^2\sum\alpha^2 - 8c^3)$ $= \frac{1}{c}(c^2 - 2c[b^2 - 2ac] + 4c^2[a^2 - 2b] - 8c^3)$ <p>= etc. as above</p>	M1 M1 M1 A1 [4]	Collecting up in terms of the symmetric fns. Use of (i)'s and (ii)'s results legitimately
(iv)	<p>One root is the product of the other two</p> $\Leftrightarrow (\alpha - 2\beta\gamma)(\beta - 2\gamma\alpha)(\gamma - 2\alpha\beta) = 0$ $\Leftrightarrow c(2a + 1)^2 = 2(b + 2c)^2$ <p>Must reason \Rightarrow and \Leftarrow explicitly (or together)</p>	B1 [1]	legitimately

Question	Answer	Marks	Notes
10 (i)		M1A1 B1 B1 B1 B1 [6]	$\frac{1}{2} + \sin \theta = 0$ when $\theta = \frac{7}{6}\pi, \frac{11}{6}\pi$ Symmetry in y -axis $(\frac{1}{2}, 0)$ on initial line Correct upper portion Correct lower portion
(ii)	$A = \left(\frac{1}{2}\right) \int_0^{2\pi} \left(\frac{1}{2} + \sin \theta\right)^2 d\theta$ $= \frac{1}{2} \int_0^{2\pi} \left(\frac{1}{4} + \sin \theta + \frac{1}{2} - \frac{1}{2} \cos 2\theta\right) d\theta$ $= \frac{1}{2} \left[\frac{3}{4}\theta - \cos \theta - \frac{1}{4} \sin 2\theta \right]_0^{2\pi}$ $= \frac{3}{4}\pi$	M1 M1 A1 A1 [4]	Penalise incorrect multiples with final A0 Double-angle formula correctly integrated 3 suitable terms

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11 (i)	$F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8$	B1 [1]	all
(ii) (a)	$p_2(x) = 1 + \frac{1}{x+1} = \frac{x+2}{x+1}$	B1	(AG)
	$p_3(x) = \frac{2x+3}{x+2}$	B1	
	$p_4(x) = \frac{3x+5}{2x+3}$	B1 [3]	
(b)	$p_n(x) = \frac{F_n x + F_{n+1}}{F_{n-1} x + F_n}$	B1	May be mentioned in later in their “round up”
	Result is true for $n = 2$ (and 3 and 4)	B1	
	Assuming $p_k(x) = \frac{F_k x + F_{k+1}}{F_{k-1} x + F_k}$ (not separate from their conjecture)		
	$p_{k+1}(x) = 1 + \frac{F_{k-1} x + F_k}{F_k x + F_{k+1}}$	M1	
	$= \frac{F_k x + F_{k+1}}{F_k x + F_{k+1}} + \frac{F_{k-1} x + F_k}{F_k x + F_{k+1}}$ $= \frac{(F_k + F_{k-1}) x + (F_k + F_{k+1})}{F_k x + F_{k+1}}$ $= \frac{F_{k+1} x + F_{k+2}}{F_k x + F_{k+1}}$	M1 A1	
	which is the required formula with $n = k + 1$. Accept this as sufficient that proof follows by induction.	[5]	Collecting coeffts. into successive Fib. terms

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12 (i)	$y = \ln\left(\tanh \frac{1}{2}x\right) \Rightarrow \frac{dy}{dx} = \frac{1}{\tanh \frac{1}{2}x} \cdot \frac{1}{2} \operatorname{sech}^2 \frac{1}{2}x$ $= \operatorname{cosech} x$	M1A1 A1 [3]	(AG)
(ii) (a)	$L_n = \int_n^{2n} \sqrt{1 + \operatorname{cosech}^2 x} \, dx$ $= \int_n^{2n} \operatorname{coth} x \, dx$ $= [\ln(\sinh x)]$ $\ln\left(\frac{\sinh 2n}{\sinh n}\right) = \ln\left(\frac{e^{2n} - e^{-2n}}{e^n - e^{-n}}\right)$ $\approx \ln\left(\frac{e^{2n}}{e^n}\right), \text{ for large } n, = \ln(e^n) = n$ <p>OR</p> $\ln\left(\frac{\sinh 2n}{\sinh n}\right) = \ln(2 \cosh n) = \ln(e^n + e^{-n})$ $\approx \ln(e^n) \text{ for large } n, = n \quad \mathbf{A1}$	M1 A1 A1 M1 A1 [5]	correct integrn. legitimately
(b)	Method (sketch or statement) to indicate that C asymptotically “merges” with the x -axis so that C is approximately a horizontal straight-line from $(n, 0)$ to $(2n, 0)$	M1 A1 [2]	legitimately
13 (i) (a)	<p>Let $y = \sec^{-1}x$, i.e. $\sec y = x$</p> $\Rightarrow \cos y = \frac{1}{x} \Rightarrow y = \cos^{-1}\left(\frac{1}{x}\right)$ <p>Then $\frac{d}{dx}(\sec^{-1}x) = \frac{d}{dx}\left(\cos^{-1}\frac{1}{x}\right)$</p> $= -\frac{1}{\sqrt{1 - (1/x)^2}} \times \frac{-1}{x^2}$ $= \frac{1}{x\sqrt{x^2 - 1}}$ <p>[Allow M1 A1 for valid non-“deduced” approaches]</p>	B1 M1 A1 [3]	(Using MF20 and the <i>Chain Rule</i>) (AG)

Question	Answer	Marks	Notes
(b)	$\int \sec^{-1} x \cdot 1 \, dx$ $= x \cdot \sec^{-1} x - \int x \cdot \frac{1}{x\sqrt{x^2-1}} \, dx$ $= [x \cdot \sec^{-1} x - \cosh^{-1} x]$	M1 A1 A1 A1 [4]	Use of integration by “parts” Condone lack of “+ C”
(ii) (a)	$\frac{1}{x\sqrt{x^2-1}} = \frac{1}{\sqrt{2}} \Rightarrow x^2(x^2-1) = 2$ $\Rightarrow x^4 - x^2 - 2 = (x^2-2)(x^2+1) = 0$ $\Rightarrow x = \sqrt{2} \quad \text{and} \quad y = \frac{1}{4}\pi$	M1 A1 A1	i.e. $P = (\sqrt{2}, \frac{1}{4}\pi)$
	<p style="text-align: center;">$Q(c, 0) \quad \sqrt{2} \quad \frac{1}{4}\pi$</p>		
	$\frac{\frac{1}{4}\pi}{\sqrt{2}-c} = \frac{1}{\sqrt{2}}$ $c = \sqrt{2} - \frac{\pi\sqrt{2}}{4}$	M1 A1 A1 [6]	or by $y - \frac{1}{4}\pi = \frac{1}{\sqrt{2}}(x - \sqrt{2})$ & $y = 0$ i.e. $Q = \left(\sqrt{2} - \frac{\pi\sqrt{2}}{4}, 0\right)$
(b)	$\text{Area } \Delta = \frac{1}{2} \times \frac{\pi\sqrt{2}}{4} \times \frac{\pi}{4} = \frac{\pi^2\sqrt{2}}{32}$ $\text{Area under curve} = \sqrt{2} \cdot \frac{\pi}{4} - \ln(1 + \sqrt{2})$ $\text{Then } R = \frac{\pi^2\sqrt{2}}{32} - \frac{\pi\sqrt{2}}{4} + \ln(1 + \sqrt{2})$ $= \ln(1 + \sqrt{2}) - \frac{\pi(8-\pi)\sqrt{2}}{32}$	B1 B1 M1 A1 [4]	using (iii)'s answer and the limits $(1, \sqrt{2})$ Difference in areas (AG)