

Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/34**

Paper 3 Further Mechanics

May/June 2026**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

This document consists of **25** printed pages.

General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: - the specific content defined in the mark scheme - the specific skills defined in the mark scheme or in the level descriptions - the standard of response required as exemplified by the standardisation scripts.
2	Examiners must award whole marks (not half marks, or other fractions).
3	Examiners must award marks positively . This means that: - marks are not deducted for errors or omissions - marks are awarded for correct/valid answers as defined in the mark scheme and also for other valid answers which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate - the quality of spelling, punctuation and grammar are judged only when the mark scheme indicates that these features are specifically assessed in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must consistently apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the full range of marks defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and must not award marks with grade thresholds or grade descriptions in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

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Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$T \cos \theta = mg$	B1	
	$T \sin \theta = \frac{m(2\sqrt{ag})^2}{a \sin \theta}$	B1	Do not accept $T \sin \theta = \frac{m(2\sqrt{ag})^2}{r}$, $r = a \sin \theta$ until both combined with r eliminated.
	$\frac{mg}{\cos \theta} \times \sin \theta = \frac{m(2\sqrt{ag})^2}{a \sin \theta}$	M1	Eliminating T from <i>their</i> dimensionally correct horizontal and vertical equations. <i>Their</i> horizontal and vertical equations can contain sin/cos mix only (therefore <i>their</i> r must be either $a \sin \theta$ or $a \cos \theta$ only).
	$\left[1 - \cos^2 \theta = 4 \cos \theta \Rightarrow\right] \cos \theta = -2 + \sqrt{5}$ ONLY	A1	WWW OE, accept AWRT 0.236 (0.2360679...). No working is required to solve the quadratic equation, but if present then it must be correct. If $\cos \theta = -2 \pm \sqrt{5}$, then $\cos \theta = -2 - \sqrt{5}$ must be rejected (but no reasoning is required for rejection).
		4	

Question	Answer	Marks	Guidance																				
2(a)	EITHER: Taking moments about AD : $\left(a^2 - \frac{1}{8}a^2\left(1 + \frac{1}{2}\right)\right)\bar{x} = a^2 \times \frac{1}{2}a - \frac{1}{8}a^2 \times \frac{1}{2}\left(a - \frac{1}{6}a \times \frac{1}{2}\right) - \frac{1}{8}a^2 \times \frac{5}{6}a$ $\left[\Rightarrow \frac{13}{16}a^2\bar{x} = \frac{1}{2}a^3 - \frac{1}{16}a^2\left(\frac{11}{12}a\right) - \frac{5}{48}a^3\right]$ Taking moments about AB : $\left(a^2 - \frac{1}{8}a^2\left(1 + \frac{1}{2}\right)\right)\bar{y} = a^2 \times \frac{1}{2}a - \frac{1}{8}a^2 \times \frac{1}{2} \times \frac{1}{6}a - \frac{1}{8}a^2 \times \frac{5}{6}a$ $\left[\Rightarrow \frac{13}{16}a^2\bar{y} = \frac{1}{2}a^3 - \frac{1}{96}a^3 - \frac{5}{48}a^3\right]$	(M1)	Dimensionally correct moments equation about AD with all terms present. At least two correct terms. For reference: <table><tr><td></td><td>Area</td><td>From AD</td><td>From AB</td></tr><tr><td>$ABCD$</td><td>a^2</td><td>$\frac{1}{2}a$</td><td>$\frac{1}{2}a$</td></tr><tr><td>BFE</td><td>$\frac{1}{8}a^2 \times \frac{1}{2}$</td><td>$a - \frac{1}{6}a \times \frac{1}{2}$</td><td>$\frac{1}{6}a$</td></tr><tr><td>$CFG$</td><td>$\frac{1}{8}a^2$</td><td>$\frac{5}{6}a$</td><td>$\frac{5}{6}a$</td></tr><tr><td>Shape $BCDEF$</td><td>$a^2 - \frac{1}{8}a^2\left(1 + \frac{1}{2}\right)$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr></table>		Area	From AD	From AB	$ABCD$	a^2	$\frac{1}{2}a$	$\frac{1}{2}a$	BFE	$\frac{1}{8}a^2 \times \frac{1}{2}$	$a - \frac{1}{6}a \times \frac{1}{2}$	$\frac{1}{6}a$	CFG	$\frac{1}{8}a^2$	$\frac{5}{6}a$	$\frac{5}{6}a$	Shape $BCDEF$	$a^2 - \frac{1}{8}a^2\left(1 + \frac{1}{2}\right)$	$\bar{x}$	$\bar{y}$
		Area	From AD	From AB																			
	$ABCD$	a^2	$\frac{1}{2}a$	$\frac{1}{2}a$																			
	BFE	$\frac{1}{8}a^2 \times \frac{1}{2}$	$a - \frac{1}{6}a \times \frac{1}{2}$	$\frac{1}{6}a$																			
	CFG	$\frac{1}{8}a^2$	$\frac{5}{6}a$	$\frac{5}{6}a$																			
Shape $BCDEF$	$a^2 - \frac{1}{8}a^2\left(1 + \frac{1}{2}\right)$	$\bar{x}$	$\bar{y}$																				
		M1	Dimensionally correct moments equation about AB with all terms present. At least two correct terms.																				
		A1	One correct unsimplified equation.																				
	$\bar{x} = \frac{5}{12}a$	A1	AG WWW, A0 if any errors seen. Allow a correct equation followed by this answer with no intermediate working for this mark.																				
	$\bar{y} = \frac{37}{78}a$	A1)	WWW but no working required. OE, answer must be exact.																				

Question	Answer	Marks	Guidance
2(a)	OR: Alternative – rectangle and two triangles.	(M1)	Dimensionally correct moments equation about AD with all terms present. At least two correct terms.
	Taking moments about AD : $\left(\frac{1}{2}a^2 + \frac{1}{8}a^2 + \frac{3}{16}a^2\right)\bar{x} = \frac{1}{2}a^2 \times \frac{1}{4}a + \frac{1}{8}a^2 \times \frac{7}{12}a + \frac{3}{16}a^2 \times \frac{3}{4}a$	M1	Dimensionally correct moments equation about AB with all terms present. At least two correct terms.
	Taking moments about AB : $\left(\frac{1}{2}a^2 + \frac{1}{8}a^2 + \frac{3}{16}a^2\right)\bar{y} = \frac{1}{2}a^2 \times \frac{1}{2}a + \frac{1}{8}a^2 \times \frac{1}{3}a + \frac{3}{16}a^2 \times \frac{1}{2}a$	A1	One correct unsimplified equation.
	$\bar{x} = \frac{5}{12}a$	A1	AG WWW, A0 if any errors seen. Allow a correct equation followed by this answer with no intermediate working for this mark.
	$\bar{y} = \frac{37}{78}a$	A1)	WWW but no working required. OE, answer must be exact.
	Available marks	5	
2(b)	$\tan \theta = \frac{\frac{1}{2}a - \frac{5}{12}a}{a - \bar{y}}$	M1	For $\tan \theta = \frac{\frac{1}{2}a - \frac{5}{12}a}{a - \bar{y}}$ or $\tan \theta = \frac{a - \bar{y}}{\frac{1}{2}a - \frac{5}{12}a}$ OE with <i>their</i> $\bar{y}$ substituted. <i>Their</i> $\bar{y}$ must be of the form ka , where $0 < k < 1$. Note that use of $\bar{y} = \frac{37}{78}a$ leads to $\tan \theta = \frac{13}{82}$.
	$\theta = 9.0^\circ$	A1	WWW, must come from a correct (but possibly inexact) value of $\bar{y} = (0.4743\dots)a$. Allow AWRT 9 (9.0085037...) or 0.157 (0.157228...) in radians.
		2	

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Question	Answer	Marks	Guidance
3(a)	$T_A + mg \cos 60 = \frac{mu^2}{a}$	M1	N2L radially at <i>A</i> (radius must be <i>a</i>) with correct number of dimensionally correct terms. Allow sin/cos mix and sign errors only. Angle must be substituted.
	$T_B - mg = \frac{m \times (2u)^2}{a}$	B1	Correct application of N2L radially at <i>B</i> . $v = 2u$ must be substituted. Radius must be <i>a</i> .
	$\frac{1}{2}m \times (2u)^2 - \frac{1}{2}mu^2 = mga(1 + \cos 60)$ $\left[\frac{3}{2}u^2 = \frac{3}{2}ga \Rightarrow u^2 = ga \right]$	*M1	Three-term energy equation (OE) for the motion at <i>A</i> and at <i>B</i> . Must be dimensionally correct. Allow sign errors and sin/cos mix only . Mass must be present. Angle must be substituted.
	$T_A + mg \cos 60 = \frac{m}{a} \times ga \quad \left[\Rightarrow T_A = \frac{1}{2}mg \right]$ or $T_B - mg = \frac{m \times (2\sqrt{ga})^2}{a} \quad \left[\Rightarrow T_B = 5mg \right]$	DM1	Substitute <i>their</i> expression for <i>u</i> (which must be in terms of <i>a</i> and possibly <i>g</i>) into either of <i>their</i> N2L equations to obtain an equation containing T_A or T_B (not both). <i>Their</i> N2L must contain the correct number of dimensionally correct terms (with sin/cos mix and sign errors only).
	$T_A = \frac{1}{2}mg$ or $T_A = 5m$ or $T_B = 5mg$ or $T_B = 50m$	A1	For either tension correct in terms of <i>m</i> (and possibly <i>g</i>) or, for example, $2T_A = 10m$.
	$k = 10$	A1	WWW, accept $T_B = 10T_A$ OE.
3(b)	[Loss in KE =] $2mga$	B1	WWW, if KE terms seen then $KE_{\text{bottom}} = 2mga$ and $KE_{\text{top}} = 0$.
		1	

Question	Answer	Marks	Guidance
4(a)	Case 1: $mu = kmv_A - mv_B$ $0.2u = v_A + v_B$ Case 2: $mu = kmv_A + mv_B$ $0.2u = v_A - v_B$	M1	PCLM: must include m (but not g). NEL: must be 0.2 multiplied by u . RHS of one equation must be the sum of v_A and v_B and the RHS of the other equation must be a difference of v_A and v_B . For example, allow for case 2: $0.2u = v_B - v_A$ for M1 .
	Case 1: $v_B = \left(\frac{0.2k-1}{k+1}\right)u$ or $v_B = \left(\frac{k-5}{6}\right)v_A$ or $6v_B = (k-5)v_A$ Case 2: $v_B = \left(\frac{1-0.2k}{k+1}\right)u$ or $v_B = \left(\frac{5-k}{6}\right)v_A$ or $6v_B = (5-k)v_A$	A1	Any equivalent (possibly unsimplified) correct expression for v_B in terms of u or a correct equation in v_A and v_B (allow with m present but not u). For reference, $v_A = \frac{6}{5(k+1)}u$.
	Case 1: $0.2k - 1 > 0 \Rightarrow k > 5$ Case 2: $1 - 0.2k < 0 \Rightarrow k > 5$	A1	AG WWW, set correct expression for $v_B > 0$ (case 1) or $v_B < 0$ (case 2) followed by $k > 5$. Accept (with equivalents for case 2), for example, $\left(\frac{0.2k-1}{k+1}\right)u > 0 \Rightarrow k > 5$ or $\frac{0.2k-1}{k+1} > 0 \Rightarrow k > 5$, OE with v_B in terms of v_A , for example, $(v_B =) \left(\frac{k-5}{6}\right)v_A > 0 \Rightarrow k > 5$. Award the final two A1 marks for $v_A = \frac{1.2u}{1+k} \Rightarrow \frac{1.2u}{1+k} - 0.2u < 0 \therefore 1+k > 6$ so $k > 5$ with the first A1 mark for $(v_B =) \frac{1.2u}{1+k} - 0.2u$. A0 for any errors seen.
		3	

Question	Answer	Marks	Guidance
4(b)	$w_B = 0.8v_B \left[= 0.8 \times \frac{0.2k-1}{k+1}u \right]$	B1	Multiply <i>their</i> expression for v_B (in any form) by ± 0.8 or for quote $w_B = \pm 0.8v_B$ provided it is clear.
	$\frac{d}{v_B} + \frac{2d}{w_B} = \frac{d}{v_A}$ $\Rightarrow \frac{d}{\left(\frac{0.2k-1}{k+1}\right)u} + \frac{2d}{\left(0.8 \times \left(\frac{0.2k-1}{k+1}\right)u\right)} = \frac{d}{\left(\frac{1.2}{k+1}\right)u}$	M1	For correctly stating or applying $\frac{d}{v_B} + \frac{2d}{w_B} = \frac{d}{v_A}$ with <i>their</i> expressions. Must be an equation in terms of speeds and distances only (not time). For reference, if correct, then may see $\frac{1}{\left(\frac{0.2k-1}{k+1}\right)} + \frac{2}{\left(0.8 \times \frac{0.2k-1}{k+1}\right)} = \frac{1}{\frac{1.2}{k+1}} \text{ OE}$ or $\frac{1}{0.2k-1} + \frac{2}{0.8 \times (0.2k-1)} = \frac{5}{6}$. Candidates may derive the result that $v_B = 3.5v_A$ (possibly leading to $\frac{6}{k-5} = \frac{2}{7}$) as part of <i>their</i> working.
	$k = 26$	A1	WWW
		3	

Question	Answer	Marks	Guidance
5(a)	The initial velocity $u > 0$ and the initial acceleration $u^2 - ku^2 < 0$.	B1	The following three parts are all required: $u > 0$ or initial velocity > 0 or in words (for example, initial velocity is positive). However, ‘velocity is > 0’ only is B0. - The expression $u^2 - ku^2$ explicitly seen (OE, for example, $-u^2(k - 1)$ not with m). $a < 0$ OE (for example, $u^2 - ku^2 < 0$, ‘the acceleration is negative’ condone lack of ‘initial’ as this is covered by seeing the correct expression).
		1	

Question	Answer	Marks	Guidance
5(b)	$mv \frac{dv}{dx} = m(v^2 - ku^2)$ or $v \frac{dv}{dx} = v^2 - ku^2$	B1	Acceleration must be $v \frac{dv}{dx}$.
	$x = \int \frac{v}{v^2 - ku^2} dv \Rightarrow x = \frac{1}{2} \ln v^2 - ku^2 + c$ or $x = -\int \frac{v}{ku^2 - v^2} dv \Rightarrow x = \frac{1}{2} \ln ku^2 - v^2 + c$	*M1	Separate variables correctly and obtain an equation of the form $x = A \ln Bv^2 + Cku^2 $ where A , B and C are non-zero constants. Or $x = A \ln(Bv + Cku) + D \ln(Ev + Fku)$ where A , B , C , D , E and F are non-zero constants, but A and D must have the same sign. Condone lack of modulus and condone no $+c$.
	$x = 0, v = u \Rightarrow 0 = \frac{1}{2} \ln u^2 - ku^2 + c$ $\Rightarrow c = -\frac{1}{2} \ln u^2 - ku^2 $ or $c = -\frac{1}{2} \ln(ku^2 - u^2)$ or $c = -\frac{1}{2} \ln ku^2 - u^2 $	DM1	Use correct initial conditions of $x = 0$ and $v = u$ correctly to find c . Note that $c = -\frac{1}{2} \ln(u^2 - ku^2)$ is M0 (as this gives an undefined value for c). ISW after seeing a correct c . Note that these values of c are dependent on the side in which the candidate added $+c$ (so may see c being the opposite sign).
	$\left[x = \frac{1}{2} \ln \left \frac{ku^2 - v^2}{u^2 - ku^2} \right , v = 0 \Rightarrow \right] x = \frac{1}{2} \ln \left(\frac{k}{k-1} \right)$ or $x = \frac{1}{2} \ln \left \frac{k}{k-1} \right $	A1	WWW, OE, for example, $\frac{1}{2} \ln \left \frac{k}{1-k} \right $, $\frac{1}{2} \ln \left(\frac{k}{ 1-k } \right)$ etc. Note: $\frac{1}{2} \ln \left(\frac{k}{1-k} \right)$ is A0 .
		4	

Question	Answer	Marks	Guidance
5(c)	$m \frac{dv}{dt} = m(v^2 - ku^2)$ or $\frac{dv}{dt} = v^2 - ku^2$	B1	Acceleration must be $\frac{dv}{dt}$.
	$t = \int \frac{1}{v^2 - ku^2} dv = \frac{1}{2u\sqrt{k}} \ln \left \frac{v - u\sqrt{k}}{v + u\sqrt{k}} \right + c$ or $t = - \int \frac{1}{ku^2 - v^2} dv = - \frac{1}{2u\sqrt{k}} \ln \left \frac{u\sqrt{k} + v}{u\sqrt{k} - v} \right + c$	M1	Separate variables correctly and obtain an equation of the form $t = \frac{K}{u\sqrt{k}} \ln \left \frac{\pm v \pm u\sqrt{k}}{\pm v \pm u\sqrt{k}} \right $ where K is a non-zero constant (or equivalent as two separate log terms). Condone lack of modulus and condone no $+c$.
	[When $t = 0$, $v = u \Rightarrow$] $c = - \frac{1}{2u\sqrt{k}} \ln \left \frac{1 - \sqrt{k}}{1 + \sqrt{k}} \right $ or $c = \frac{1}{2u\sqrt{k}} \ln \left \frac{\sqrt{k} + 1}{\sqrt{k} - 1} \right $	A1	OE, for example, $c = - \frac{1}{2u\sqrt{k}} \ln \left(\frac{\sqrt{k} - 1}{\sqrt{k} + 1} \right)$ or $- \frac{1}{2u\sqrt{k}} \ln \left(\frac{ 1 - \sqrt{k} }{1 + \sqrt{k}} \right)$ but $- \frac{1}{2u\sqrt{k}} \ln \left(\frac{1 - \sqrt{k}}{1 + \sqrt{k}} \right)$ (or any other undefined expression) is A0 . Award even if u appears in all terms in the fraction in the argument of the natural logarithm. Note that these values of c are dependent on the side in which the candidate added $+c$ (so may see c being the opposite sign).
	$\left[t = \frac{1}{2u\sqrt{k}} \ln \left \frac{v - u\sqrt{k}}{v + u\sqrt{k}} \right - \frac{1}{2u\sqrt{k}} \ln \left \frac{1 - \sqrt{k}}{1 + \sqrt{k}} \right \text{ so when } v = 0 \Rightarrow \right]$ $[t =] \frac{1}{2u\sqrt{k}} \ln \left(\frac{\sqrt{k} + 1}{\sqrt{k} - 1} \right) \text{ or } - \frac{1}{2u\sqrt{k}} \ln \left(\frac{\sqrt{k} - 1}{\sqrt{k} + 1} \right) \text{ or } \frac{1}{2u\sqrt{k}} \ln \left(\frac{\sqrt{k} + k}{k - \sqrt{k}} \right)$	A1	WWW OE, for example, $- \frac{1}{2u\sqrt{k}} \ln \left(\frac{ 1 - \sqrt{k} }{1 + \sqrt{k}} \right)$ but A0 for $- \frac{1}{2u\sqrt{k}} \ln \left(\frac{1 - \sqrt{k}}{1 + \sqrt{k}} \right)$ etc. Must have scored the previous A1 mark. Allow u in all terms in the fraction in the argument of the natural logarithm.
		4	

Question	Answer	Marks	Guidance
6(a)	$d = \frac{1}{2}(g \sin \alpha) \times T^2$ or $\frac{1}{2}mv^2 = mgd \sin \alpha \Rightarrow v^2 = 2gd \sin \alpha$ and $v = (g \sin \alpha)T \Rightarrow \sqrt{2gd \sin \alpha} = (g \sin \alpha)T$	*B1	Throughout this part allow t instead of T for full marks. First line of working must be a direct substitution into $s = ut + \frac{1}{2}at^2$ (although $u = 0$ can be implied). Therefore, if the first line of working is $2d = g \sin \alpha \times T^2$ then award B0 . If working backwards from AG this is B0 . Energy approach – must get to a correct equation in T for B1 .
	$2d = g \sin \alpha \times T^2 \Rightarrow T = \sqrt{\frac{2d}{g \sin \alpha}}$ or $T = \frac{\sqrt{2gd \sin \alpha}}{g \sin \alpha} = \sqrt{\frac{2gd \sin \alpha}{g^2 \sin^2 \alpha}} = \sqrt{\frac{2d}{g \sin \alpha}}$	DB1	AG, at least one line of working from $d = \frac{1}{2}(g \sin \alpha) \times T^2$ to given answer OE. Allow $d = \frac{1}{2}aT^2$ with one line of working leading to $T = \sqrt{\frac{2d}{a}}$ and then given answer provided $a = g \sin \alpha$ is explicitly stated. B1 B0 for $a = g \sin \alpha$, therefore $d = \frac{1}{2}aT^2 \Rightarrow T = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2d}{g \sin \alpha}}$. Energy: $T = \frac{\sqrt{2gd \sin \alpha}}{g \sin \alpha} = \sqrt{\frac{2d}{g \sin \alpha}}$ is B0 .
		2	

Question	Answer	Marks	Guidance
6(b)	EITHER: $d \cos \alpha = uT \cos \alpha \quad [\Rightarrow d = uT] \text{ or } d \cos \alpha = u \sqrt{\frac{2d}{g \sin \alpha}} \cos \alpha$	(*B1)	Throughout this part allow t instead of T for full marks. Stating $d = uT$ implies this mark.
	$-h = uT \sin(2\alpha - \alpha) - \frac{1}{2}gT^2$ or $-h = u \times \sqrt{\frac{2d}{g \sin \alpha}} \sin(2\alpha - \alpha) - \frac{1}{2}g \left(\sqrt{\frac{2d}{g \sin \alpha}} \right)^2$	*M1	Applying $s = ut + \frac{1}{2}at^2$ (or other complete method) to find the vertical distance travelled by P . Allow $\sin \alpha$ for $\sin(2\alpha - \alpha)$. Allow y for the vertical height but not $\pm d$. Allow sign errors only.
	$-d \sin \alpha = u \times \sqrt{\frac{2d}{g \sin \alpha}} \sin \alpha - \frac{1}{2}g \left(\sqrt{\frac{2d}{g \sin \alpha}} \right)^2$ or $-uT \sin \alpha = uT \sin \alpha - \frac{1}{2}gT^2$ or $-d \sin \alpha = d \sin \alpha - \frac{1}{2}gT^2$	A1	OE, obtaining a second correct equation in two of d , u , and T . Can be implied by $T = \frac{4u \sin \alpha}{g}$ or $T^2 = \frac{4d \sin \alpha}{g}$.
	$\left[\frac{4d \sin \alpha}{g} = \left(\sqrt{\frac{2d}{g \sin \alpha}} \right)^2 \Rightarrow \right] 4 \sin \alpha = \frac{2}{\sin \alpha} \text{ or } -\sin \alpha = \sin \alpha - \frac{1}{\sin \alpha}$	DM1	Eliminate all given unknowns to obtain an equation in α only.
	$\alpha = 45^\circ$	A1)	OE, for example, $\frac{1}{4}\pi$ or AWRT 0.785 (0.7853981...) radians.

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Question	Answer	Marks	Guidance
6(b)	OR: Alternative approach 1 – considering the motion of P parallel to the slope $d = (u \cos 2\alpha)T + \frac{1}{2}(g \sin \alpha)T^2$	(*M1)	Throughout this part allow t instead of T for full marks. Applying $s = ut + \frac{1}{2}at^2$ parallel to the slope. Allow sign errors only . Must be d on the LHS.
		A1	Fully correct equation.
	$d = (u \cos 2\alpha) \times \sqrt{\frac{2d}{g \sin \alpha}} + \frac{1}{2} \times (g \sin \alpha) \times \left(\sqrt{\frac{2d}{g \sin \alpha}} \right)^2$	A1	Eliminate one of T , d or u to obtain a correct equation in the other two variables, for example, $\frac{T^2 g \sin \alpha}{2} = (u \cos 2\alpha)T + \frac{1}{2} \times (g \sin \alpha) \times T^2.$
	$[uT \cos 2\alpha = 0 \Rightarrow] \cos 2\alpha = 0$	DM1	Eliminate all given unknowns to obtain an equation in α only.
	$\alpha = 45^\circ$	A1)	OE, for example, $\frac{1}{4}\pi$ or AWR 0.785 (0.7853981...) radians.

Question	Answer	Marks	Guidance
6(b)	OR: Alternative approach 2 – equation of the trajectory $d \cos \alpha = uT \cos \alpha \quad [\Rightarrow d = uT] \quad \text{or} \quad d \cos \alpha = u \sqrt{\frac{2d}{g \sin \alpha}} \cos \alpha$	(*B1)	Throughout this part allow t instead of T for full marks. Stating $d = uT$ implies this mark. Or for another correct equation in at least two of d , u and T .
	$-d \sin \alpha = d \cos \alpha \times \tan \alpha - \frac{g(d \cos \alpha)^2}{2u^2 \cos^2 \alpha}$	*M1	Applying the equation of the trajectory. Allow sign errors only . LHS must be $\pm d \sin \alpha$.
	$[\Rightarrow gd = 4u^2 \sin \alpha]$	A1	Fully correct unsimplified equation.
	$[4u^2 \sin^2 \alpha = 2u^2 \Rightarrow] \quad 2 \sin^2 \alpha = 1$	DM1	Eliminate all given unknowns to obtain an equation in α only.
	$\alpha = 45^\circ$	A1)	OE, for example, $\frac{1}{4}\pi$ or AWRT 0.785 (0.7853981...) radians.
	Available marks	5	

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Question	Answer	Marks	Guidance
7(a)	$R = mg \times \frac{12}{13} \quad \left[= \frac{120}{13} m \right]$	B1	Correct expression for the normal contact force. Allow 23 or better (22.61986...) for angle. Allow $R = mg \cos\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$. Angle must be substituted.
	$T - \mu \times \frac{12}{13} mg - \frac{5}{13} mg = m \times \frac{5}{4} g$	M1	Use of N2L parallel to the slope. Must contain the correct number of dimensionally correct relevant terms. Allow sin/cos mix, allow sign errors. Must include a component of the weight not mass. Allow with T or <i>their</i> possibly incorrect T (but all other values must be substituted). Normal contact force must be either $\frac{12}{13} mg$ or $\frac{5}{13} mg$ (OE, using an angle of 23 or better).
	$T = \frac{6mg \times 3a}{8a} \quad \left[= \frac{9}{4} mg \right]$	B1	Correct unsimplified expression for T using Hooke's law.
	$\left[\frac{9}{4} mg - \mu \times \frac{12}{13} mg - \frac{5}{13} mg = m \times \frac{5}{4} g \right]$ $\mu = \frac{2}{3}$	A1	Allow AWRT 0.667 (but not 0.666).
		4	

Question	Answer	Marks	Guidance
7(b)	EITHER: METHOD 1 (SEE SC ON PAGE 25 FOR THOSE WHO ASSUME THAT P COMES TO REST AFTER THE STRING IS SLACK) $\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 \quad \left[= \frac{27}{8} mga \right]$	(*B1)	B0 for $\frac{1}{2} \times \frac{6mg}{8a} \times 3a^2$ unless recovered.
	$\frac{1}{2} \times \frac{6mg(3a)^2}{8a} = \frac{1}{2} \times \frac{6mg}{8a} \times (3a-d)^2 + \frac{5}{13} \times mgd + \frac{2}{3} \left(\frac{12}{13} mgd \right)$ or $\frac{1}{2} \times \frac{6mg(3a)^2}{8a} = \frac{1}{2} \times \frac{6mg}{8a} \times e^2 + \frac{5}{13} mg(3a-e) + \frac{2}{3} \left(\frac{12}{13} mg(3a-e) \right)$ or $\frac{1}{2} \times \frac{6mg(3a)^2}{8a} = \frac{1}{2} \times \frac{6mg(x-8a)^2}{8a} + \frac{5}{13} mg(11a-x) + \frac{2}{3} \left(\frac{12}{13} mg(11a-x) \right)$	*M1	Work-energy equation using <i>their</i> μ with the correct number of dimensionally correct relevant terms. Allow sign errors. Allow sin/cos mix on the GPE and work done by friction. Final EPE must be dimensionally correct, and its extension X must be consistent with <i>their</i> GPE and work done by friction terms, but allow other slips in <i>their</i> EPE term. If <i>their</i> X in EPE implies that $X < 0$, then M0 .
		A1FT	Fully correct equation. FT <i>their</i> value of μ .
	$\frac{3d^2}{8a} - \frac{5}{4}d = 0 \Rightarrow d = \frac{10a}{3} \quad [> 3a \text{ so } P \text{ moving when the string is slack}]$ or $(3e+a)(e-3a) = 0 \Rightarrow e = -\frac{1}{3}a$	A1	Correctly obtaining $d = \frac{10a}{3}$ or $e = -\frac{1}{3}a$ or $x = \frac{23}{3}a$ from $(3x-23a)(x-11a) = 0$
	$\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 = mgd \times \frac{5}{13} + \frac{2}{3} \times \left(\frac{12}{13} mgd \right)$ or $\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 = mgd$ or $\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 = mg(3a-e) \times \frac{5}{13} + \frac{2}{3} \times \left(\frac{12}{13} mg(3a-e) \right)$ OE	DM1	OE, work-energy equation with correct number of dimensionally correct relevant terms. Allow sign errors. Allow sin/cos mix on the GPE and work done by friction (but these two terms must contain a component of weight). Must be using correct EPE at this stage.
	$\left[d = \frac{27}{8}a \Rightarrow OC = 11a - \frac{27}{8}a = \right] \quad \frac{61}{8}a$	A1)	WWW 7.625a (condone 7.63a). For reference, $e = -\frac{3}{8}a \Rightarrow OC = 11a - 3a - \frac{3}{8}a = \frac{61}{8}a$.

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Question	Answer	Marks	Guidance
7(b)	OR: METHOD 2 (Showing KE or velocity is > 0 when the string goes slack) $\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 \quad \left[= \frac{27}{8} mga \right]$	(*B1)	Correct unsimplified expression for the initial EPE.
	$\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 = KE_{\text{slack}} + 3a \times mg \times \frac{5}{13} + \frac{2}{3} \times \left(\frac{12}{13} mg \times 3a \right)$ or $\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 = KE_{\text{slack}} + 3mga$	*M1	Work-energy equation using <i>their</i> μ with the correct number of dimensionally correct relevant terms. Allow sign errors. Allow sin/cos mix on the GPE and work done by friction (but these two terms must contain a component of weight and the correct distance of $3a$). EPE must be of the form $k_1(3a)^2$ where k_1 is dimensionally correct, but allow other slips. May see $KE_{\text{slack}} = \frac{1}{2} mv^2$.
		A1FT	Fully correct equation. FT <i>their</i> value of μ .
	$KE = \frac{3}{8} mga$ or $v^2 = \frac{3}{4} ga$ [> 0 so P is moving when the string is slack].	A1	WWW, correct expression for the KE when the string goes slack OE. No comment required that this is positive (OE).
	$\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 = mgd \times \frac{5}{13} + \frac{2}{3} \times \left(\frac{12}{13} mgd \right)$ or $\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 = mgd$	DM1	Work-energy equation with correct number of dimensionally correct relevant terms – allow sign errors. Allow sin/cos mix on the GPE and work done by friction (but these two terms must contain a component of weight). Must be using correct EPE at this stage.
	$\left[d = \frac{27}{8} a \Rightarrow OC = 11a - \frac{27}{8} a = \right] \quad \frac{61}{8} a$	A1)	WWW 7.625a (condone 7.63a).

Question	Answer	Marks	Guidance
7(b)	OR: SC FOR THOSE WHO <u>ASSUME</u> THAT P COMES TO REST AFTER THE STRING IS SLACK $\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 \quad \left[= \frac{27}{8} mga \right]$	(*B1)	B0 for $\frac{1}{2} \times \frac{6mg}{8a} \times 3a^2$ unless recovered.
	$\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 = mgd \times \frac{5}{13} + \frac{2}{3} \times \left(\frac{12}{13} mgd \right)$ or $\frac{1}{2} \times \frac{6mg}{8a} \times (3a)^2 = mgd$	DM1	OE, work-energy equation with correct number of dimensionally correct relevant terms. Allow sign errors. Allow sin/cos mix on the GPE and work done by friction (but these two terms must contain a component of weight). Must be using the correct expression for EPE and <i>their</i> μ from 7(a).
	$\left[d = \frac{27}{8} a \Rightarrow OC = 11a - \frac{27}{8} a = \right] \quad \frac{61}{8} a$	A1	7.625a (condone 7.63a) Maximum of three marks for the SC.
	The maximum number of marks available to candidates who assume that P comes to rest after the string is slack with no further work is 3 marks as above. However, full marks available for a candidate who explicitly shows why P is still in motion when the string goes slack. An example of such an approach is as follows.		
	[Sum of work done by friction to travel $3a$ up the plane and the GPE is given by] $mg \times 3a \times \frac{5}{13} + \frac{2}{3} \times \left(\frac{12}{13} mg \times 3a \right) = 3mga$	M1	For sum of the two terms with sin/cos mix only and <i>their</i> μ .
		A1FT	FT <i>their</i> μ .
	[As] $\frac{27}{8} mga > 3mga$, [P is still in motion.]	A1)	Correct comparison with the EPE.
	Available marks	6	