

Past Years: Chapter 5 Normal Distribution

May/June 2002

- 6 (i) In a normal distribution with mean μ and standard deviation σ , $P(X > 3.6) = 0.5$ and $P(X > 2.8) = 0.6554$. Write down the value of μ , and calculate the value of σ . [4]
- (ii) If four observations are taken at random from this distribution, find the probability that at least two observations are greater than 2.8. [4]
- 7 (i) A garden shop sells polyanthus plants in boxes, each box containing the same number of plants. The number of plants per box which produce yellow flowers has a binomial distribution with mean 11 and variance 4.95.
- (a) Find the number of plants per box. [4]
- (b) Find the probability that a box contains exactly 12 plants which produce yellow flowers. [2]
- (ii) Another garden shop sells polyanthus plants in boxes of 100. The shop's advertisement states that the probability of any polyanthus plant producing a pink flower is 0.3. Use a suitable approximation to find the probability that a box contains fewer than 35 plants which produce pink flowers. [4]

May/June 2003

- 3 (i) The height of sunflowers follows a normal distribution with mean 112 cm and standard deviation 17.2 cm. Find the probability that the height of a randomly chosen sunflower is greater than 120 cm. [3]
- (ii) When a new fertiliser is used, the height of sunflowers follows a normal distribution with mean 115 cm. Given that 80% of the heights are now greater than 103 cm, find the standard deviation. [3]
- 4 Kamal has 30 hens. The probability that any hen lays an egg on any day is 0.7. Hens do not lay more than one egg per day, and the days on which a hen lays an egg are independent.
- (i) Calculate the probability that, on any particular day, Kamal's hens lay exactly 24 eggs. [2]
- (ii) Use a suitable approximation to calculate the probability that Kamal's hens lay fewer than 20 eggs on any particular day. [5]

May/June 2004

- 4 Melons are sold in three sizes: small, medium and large. The weights follow a normal distribution with mean 450 grams and standard deviation 120 grams. Melons weighing less than 350 grams are classified as small.
- (i) Find the proportion of melons which are classified as small. [3]
- (ii) The rest of the melons are divided in equal proportions between medium and large. Find the weight above which melons are classified as large. [5]
- 7 A shop sells old video tapes, of which 1 in 5 on average are known to be damaged.
- (i) A random sample of 15 tapes is taken. Find the probability that at most 2 are damaged. [3]
- (ii) Find the smallest value of n if there is a probability of at least 0.85 that a random sample of n tapes contains at least one damaged tape. [3]
- (iii) A random sample of 1600 tapes is taken. Use a suitable approximation to find the probability that there are at least 290 damaged tapes. [5]

May/June 2005

- 1 It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, find the probability that fewer than 165 people in the sample are overweight. [5]
- 3 A fair dice has four faces. One face is coloured pink, one is coloured orange, one is coloured green and one is coloured black. Five such dice are thrown and the number that fall on a green face are counted. The random variable X is the number of dice that fall on a green face.
- (i) Show that the probability of 4 dice landing on a green face is 0.0146, correct to 4 decimal places. [2]
- (ii) Draw up a table for the probability distribution of X , giving your answers correct to 4 decimal places. [5]
- 6 Tyre pressures on a certain type of car independently follow a normal distribution with mean 1.9 bars and standard deviation 0.15 bars.
- (i) Find the probability that all four tyres on a car of this type have pressures between 1.82 bars and 1.92 bars. [5]
- (ii) Safety regulations state that the pressures must be between $1.9 - b$ bars and $1.9 + b$ bars. It is known that 80% of tyres are within these safety limits. Find the safety limits. [3]

May/June 2006

- 3 The lengths of fish of a certain type have a normal distribution with mean 38 cm. It is found that 5% of the fish are longer than 50 cm.
- (i) Find the standard deviation. [3]
- (ii) When fish are chosen for sale, those shorter than 30 cm are rejected. Find the proportion of fish rejected. [3]
- (iii) 9 fish are chosen at random. Find the probability that at least one of them is longer than 50 cm. [2]
- 7 A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.
- (i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. [4]
- (ii) A random sample of 200 adults was taken. Using a suitable approximation, find the probability that more than 155 wore a watch on their left wrist. [5]

May/June 2007

- 3 (a) The random variable X is normally distributed. The mean is twice the standard deviation. It is given that $P(X > 5.2) = 0.9$. Find the standard deviation. [4]
- (b) A normal distribution has mean μ and standard deviation σ . If 800 observations are taken from this distribution, how many would you expect to be between $\mu - \sigma$ and $\mu + \sigma$? [3]
- 6 The probability that New Year's Day is on a Saturday in a randomly chosen year is $\frac{1}{7}$.
- (i) 15 years are chosen randomly. Find the probability that at least 3 of these years have New Year's Day on a Saturday. [4]
- (ii) 56 years are chosen randomly. Use a suitable approximation to find the probability that more than 7 of these years have New Year's Day on a Saturday. [5]

May/June 2008

- 4 In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.
- (i) Find the value of μ . [4]
- In another country the standard deviation of the time taken for the infection to clear up is the same as in part (i), but the mean is 6.5 days. The time taken is normally distributed.
- (ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up. [3]
- 7 A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.
- (i) The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number. [3]
- (ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5. [3]
- (iii) The die is thrown 90 times. Using an appropriate approximation, find the probability that a 5 is thrown more than 60 times. [5]

May/June 2009

- 3 On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars.
- (i) A random sample of 11 vehicles is taken. Find the probability that fewer than 3 are buses. [3]
- (ii) A random sample of 125 vehicles is now taken. Using a suitable approximation, find the probability that more than 73 are cars. [5]

Oct/Nov 2001

- 5 The waiting time in a doctor's surgery is normally distributed with mean 15 minutes and standard deviation 4.2 minutes.
- (i) Find the probability that a patient has to wait less than 10 minutes to see the doctor. [3]
- (ii) 10% of people wait longer than T minutes. Find T . [3]
- (iii) In a given week, 200 people attend the surgery. Estimate the number of these who wait more than 20 minutes. [3]
- 6 65% of all watches sold by a shop have a digital display and 35% have an analog display.
- (i) Find the probability that, out of the next 12 customers who buy a watch, fewer than 10 choose one with a digital display. [4]
- (ii) Use a suitable approximation to find the probability that, out of the next 120 customers who buy a watch, fewer than 70 choose one with a digital display. [5]

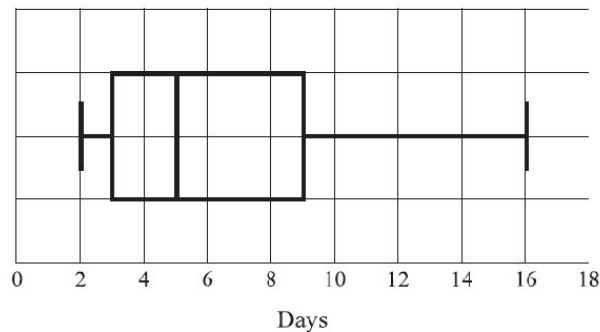
Oct/Nov 2002

- 3 The distance in metres that a ball can be thrown by pupils at a particular school follows a normal distribution with mean 35.0 m and standard deviation 11.6 m.
- (i) Find the probability that a randomly chosen pupil can throw a ball between 30 and 40 m. [3]
- (ii) The school gives a certificate to the 10% of pupils who throw further than a certain distance. Find the least distance that must be thrown to qualify for a certificate. [3]

- 6 (i) A manufacturer of biscuits produces 3 times as many cream ones as chocolate ones. Biscuits are chosen randomly and packed into boxes of 10. Find the probability that a box contains equal numbers of cream biscuits and chocolate biscuits. [2]
- (ii) A random sample of 8 boxes is taken. Find the probability that exactly 1 of them contains equal numbers of cream biscuits and chocolate biscuits. [2]
- (iii) A large box of randomly chosen biscuits contains 120 biscuits. Using a suitable approximation, find the probability that it contains fewer than 35 chocolate biscuits. [5]

Oct/Nov 2003

- 3 In a normal distribution, 69% of the distribution is less than 28 and 90% is less than 35. Find the mean and standard deviation of the distribution. [6]
- 7 The length of time a person undergoing a routine operation stays in hospital can be modelled by a normal distribution with mean 7.8 days and standard deviation 2.8 days.
- (i) Calculate the proportion of people who spend between 7.8 days and 11.0 days in hospital. [4]
- (ii) Calculate the probability that, of 3 people selected at random, exactly 2 spend longer than 11.0 days in hospital. [2]
- (iii) A health worker plotted a box-and-whisker plot of the times that 100 patients, chosen randomly, stayed in hospital. The result is shown below.



State with a reason whether or not this agrees with the model used in parts (i) and (ii). [2]

Oct/Nov 2004

- 5 The length of Paulo's lunch break follows a normal distribution with mean μ minutes and standard deviation 5 minutes. On one day in four, on average, his lunch break lasts for more than 52 minutes.
- (i) Find the value of μ . [3]
- (ii) Find the probability that Paulo's lunch break lasts for between 40 and 46 minutes on every one of the next four days. [4]
- 7 (i) State two conditions which must be satisfied for a situation to be modelled by a binomial distribution. [2]

In a certain village 28% of all cars are made by Ford.

- (ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford. [4]
- (iii) A random sample of 50 cars in the village is taken. Estimate, using a normal approximation, the probability that more than 18 cars are made by Ford. [4]

Oct/Nov 2005

- 7 In tests on a new type of light bulb it was found that the time they lasted followed a normal distribution with standard deviation 40.6 hours. 10% lasted longer than 5130 hours.
- (i) Find the mean lifetime, giving your answer to the nearest hour. [3]
 - (ii) Find the probability that a light bulb fails to last for 5000 hours. [3]
 - (iii) A hospital buys 600 of these light bulbs. Using a suitable approximation, find the probability that fewer than 65 light bulbs will last longer than 5130 hours. [4]

Oct/Nov 2006

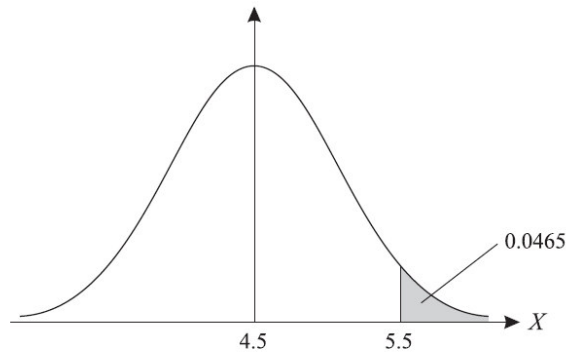
- 5 (i) Give an example of a variable in real life which could be modelled by a normal distribution. [1]
- (ii) The random variable X is normally distributed with mean μ and variance 21.0. Given that $P(X > 10.0) = 0.7389$, find the value of μ . [3]
- (iii) If 300 observations are taken at random from the distribution in part (ii), estimate how many of these would be greater than 22.0. [4]
- 7 A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are
- (i) equal numbers of large and small bands, [2]
 - (ii) more than 17 small bands. [3]

An office pack contains 150 elastic bands.

- (iii) Using a suitable approximation, calculate the probability that the number of small bands in the office pack is between 88 and 97 inclusive. [6]

Oct/Nov 2007

4



The random variable X has a normal distribution with mean 4.5. It is given that $P(X > 5.5) = 0.0465$ (see diagram).

- (i) Find the standard deviation of X . [3]
- (ii) Find the probability that a random observation of X lies between 3.8 and 4.8. [4]

- 6 On any occasion when a particular gymnast performs a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions.
- (i) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7. [2]
 - (ii) On one day she performs the routine 50 times. Use a suitable approximation to estimate the probability that she will perform the routine correctly on fewer than 29 occasions. [5]
 - (iii) On another day she performs the routine n times. Find the smallest value of n for which the expected number of correct performances is at least 8. [2]

Oct/Nov 2008

- 2 On a production line making toys, the probability of any toy being faulty is 0.08. A random sample of 200 toys is checked. Use a suitable approximation to find the probability that there are at least 15 faulty toys. [5]
- 3 (i) The daily minimum temperature in degrees Celsius ($^{\circ}\text{C}$) in January in Ottawa is a random variable with distribution $N(-15.1, 62.0)$. Find the probability that a randomly chosen day in January in Ottawa has a minimum temperature above 0°C . [3]
- (ii) In another city the daily minimum temperature in $^{\circ}\text{C}$ in January is a random variable with distribution $N(\mu, 40.0)$. In this city the probability that a randomly chosen day in January has a minimum temperature above 0°C is 0.8888. Find the value of μ . [3]

Oct/Nov 2009/11

- 3 The times for a certain car journey have a normal distribution with mean 100 minutes and standard deviation 7 minutes. Journey times are classified as follows:
- 'short' (the shortest 33% of times),
 - 'long' (the longest 33% of times),
 - 'standard' (the remaining 34% of times).
- (i) Find the probability that a randomly chosen car journey takes between 85 and 100 minutes. [3]
 - (ii) Find the least and greatest times for 'standard' journeys. [4]
- 6 A box contains 4 pears and 7 oranges. Three fruits are taken out at random and eaten. Find the probability that
- (i) 2 pears and 1 orange are eaten, in any order, [3]
 - (ii) the third fruit eaten is an orange, [3]
 - (iii) the first fruit eaten was a pear, given that the third fruit eaten is an orange. [3]
- There are 121 similar boxes in a warehouse. One fruit is taken at random from each box.
- (iv) Using a suitable approximation, find the probability that fewer than 39 are pears. [5]

Oct/Nov 2009/12

- 7 The weights, X grams, of bars of soap are normally distributed with mean 125 grams and standard deviation 4.2 grams.
- (i) Find the probability that a randomly chosen bar of soap weighs more than 128 grams. [3]
 - (ii) Find the value of k such that $P(k < X < 128) = 0.7465$. [4]
 - (iii) Five bars of soap are chosen at random. Find the probability that more than two of the bars each weigh more than 128 grams. [4]

May/June 2010/61

- 3 The random variable X is the length of time in minutes that Jannon takes to mend a bicycle puncture. X has a normal distribution with mean μ and variance σ^2 . It is given that $P(X > 30.0) = 0.1480$ and $P(X > 20.9) = 0.6228$. Find μ and σ . [5]
- 5 In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.
- (i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones. [2]
- (ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to find the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones. [1]
- (iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones. [5]

May/June 2010/62

- 2 The lengths of new pencils are normally distributed with mean 11 cm and standard deviation 0.095 cm.
- (i) Find the probability that a pencil chosen at random has a length greater than 10.9 cm. [2]
- (ii) Find the probability that, in a random sample of 6 pencils, at least two have lengths less than 10.9 cm. [3]
- 4 The random variable X is normally distributed with mean μ and standard deviation σ .
- (i) Given that $5\sigma = 3\mu$, find $P(X < 2\mu)$. [3]
- (ii) With a different relationship between μ and σ , it is given that $P(X < \frac{1}{3}\mu) = 0.8524$. Express μ in terms of σ . [3]

May/June 2010/63

- 7 The heights that children of a particular age can jump have a normal distribution. On average, 8 children out of 10 can jump a height of more than 127 cm, and 1 child out of 3 can jump a height of more than 135 cm.
- (i) Find the mean and standard deviation of the heights the children can jump. [5]
- (ii) Find the probability that a randomly chosen child will **not** be able to jump a height of 145 cm. [3]
- (iii) Find the probability that, of 8 randomly chosen children, at least 2 will be able to jump a height of more than 135 cm. [3]

Oct/Nov 2010/61

- 2 On average, 2 apples out of 15 are classified as being underweight. Find the probability that in a random sample of 200 apples, the number of apples which are underweight is more than 21 and less than 35. [5]
- 3 The times taken by students to get up in the morning can be modelled by a normal distribution with mean 26.4 minutes and standard deviation 3.7 minutes.
- (i) For a random sample of 350 students, find the number who would be expected to take longer than 20 minutes to get up in the morning. [3]
- (ii) 'Very slow' students are students whose time to get up is more than 1.645 standard deviations above the mean. Find the probability that fewer than 3 students from a random sample of 8 students are 'very slow'. [4]

Oct/Nov 2010/62

- 5 The distance the Zotoc car can travel on 20 litres of fuel is normally distributed with mean 320 km and standard deviation 21.6 km. The distance the Ganmor car can travel on 20 litres of fuel is normally distributed with mean 350 km and standard deviation 7.5 km. Both cars are filled with 20 litres of fuel and are driven towards a place 367 km away.
- (i) For each car, find the probability that it runs out of fuel before it has travelled 367 km. [3]
 - (ii) The probability that a Zotoc car can travel at least $(320 + d)$ km on 20 litres of fuel is 0.409. Find the value of d . [4]
- 6 (i) State three conditions that must be satisfied for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

- (ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]
- (iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days. [5]

Oct/Nov 2010/63

- 1 Name the distribution and suggest suitable numerical parameters that you could use to model the weights in kilograms of female 18-year-old students. [2]
- 7 The times spent by people visiting a certain dentist are independent and normally distributed with a mean of 8.2 minutes. 79% of people who visit this dentist have visits lasting less than 10 minutes.
- (i) Find the standard deviation of the times spent by people visiting this dentist. [3]
 - (ii) Find the probability that the time spent visiting this dentist by a randomly chosen person deviates from the mean by more than 1 minute. [3]
 - (iii) Find the probability that, of 6 randomly chosen people, more than 2 have visits lasting longer than 10 minutes. [3]
 - (iv) Find the probability that, of 35 randomly chosen people, fewer than 16 have visits lasting less than 8.2 minutes. [5]