WJEC
CBAC

GCE EXAMINERS' REPORTS

MATHEMATICS (C1 - C4 and FP1 - FP3)
AS/Advanced

SUMMER 2011

Statistical Information

This booklet contains summary details for each unit: number entered; maximum mark available; mean mark achieved; grade ranges. *N.B. These refer to 'raw marks' used in the initial assessment, rather than to the uniform marks reported when results are issued.*

Annual Statistical Report

The annual *Statistical Report* (issued in the second half of the Autumn Term) gives overall outcomes of all examinations administered by WJEC.

MATHEMATICS

General Certificate of Education

Summer 2011

Advanced Subsidiary/Advanced

Principal Examiner: Dr. E. Read

Unit Statistics

The following statistics include all candidates entered for the unit, whether or not they 'cashed in' for an award. The attention of centres is drawn to the fact that the statistics listed should be viewed strictly within the context of this unit and that differences will undoubtedly occur between one year and the next and also between subjects in the same year.

Unit	Entry	Max Mark	Mean Mark
C1	3475	75	39.7

Grade Ranges

A	52
B	45
C	38
D	31
E	25

N.B. The marks given above are raw marks and not uniform marks.

C1

General Comments

Performance on this year's paper was similar to that on the corresponding paper last year. Candidates found the majority of questions accessible; it was questions 5(*a*), 7(*b*) and 10 which caused most problems.

Individual Questions

Q.1 It was only the very final part of this question which caused any difficulty. Relatively few candidates were able to note that the equation of *AC* was $x = 3$.

Q.2 Although part (*a*) took a slightly different format to what is usually the case, most candidates were able to earn full marks, as was also the case in part (*b*).

Q.3 The majority of candidates got full marks on this question.

Q.4 Even though the leading coefficient of the given expression was negative, most candidates were able to get at least one of the constants *a*, *b* correct. Not all, however, realised that the graph required was that of a negative quadratic.

Q.5 Although most candidates were able to start part (*a*) correctly, many were then unable to rearrange their equation in the form $x^2 + (4k + 2)x + (7 - k) = 0$ and thus made little progress. Part (*b*) was also disappointing and fewer candidates than usual seemed to get full marks on this part.

Q.6 Both parts were generally well answered. Differentiation from first principles does seem to be improving but some candidates still lost the final mark due to a mathematically incorrect statement.

Q.7 Most candidates got full marks on part (*a*). However, in part (*b*), many were unable to find correct expressions for the coefficients of x and x^2 and relatively few got the final correct answer of $n = 41$.

Q.8 Even though part (*a*) involved setting up two simultaneous equations in *p* and *q*, it was usually only arithmetic errors which prevented candidates from getting full marks. In part (*b*), not all candidates realised that they already knew that $(x + 2)$ was one of the factors of the given expression.

Q.9 Almost all candidates got part (*a*) correct and a majority also realised that the expression required in part (*b*) was of the form $y = rf(x)$ with *r* negative.

Q.10 This was a disappointing question. Many, although not all candidates, were able to earn both marks in part (*a*). Far fewer, however, realised that part (*b*) involved using differentiation to find the maximum value of an expression and only a minority were able to get full marks on this question

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Unit	Entry	Max Mark	Mean Mark
C2	4386	75	43.0

Grade Ranges

A	59
B	51
C	43
D	36
E	29

N.B. The marks given above are raw marks and not uniform marks.

C2

General Comments

Candidates found this year's paper to be somewhat easier than the corresponding paper last year. Almost all questions were well answered and it was only in question 7 where general problems arose, usually due to the poor manipulation of logarithms.

Individual questions

Q.1 The question on the Trapezium Rule caused very few problems, as is generally the case.

Q.2 Part (*a*) was well answered but in part (*b*), some candidates missed the root corresponding to $2x - 35° = -27°$. Most candidates tried to rewrite part (*c*) in terms of $\tan \phi$ but many lost marks due to poor algebraic manipulation.

Q.3 In both parts (*a*) and (*b*), most candidates who chose the correct formula were usually able to earn full marks.

Q.4 There were few problems in part (*a*) but in part (*b*), many candidates managed to show that $d = 9$ but were then unable to find the value of the $(p + 7)$ th term.

Q.5 Part (*a*) was well answered. In part (*b*), some candidates made arithmetic errors in calculating the values of r and a, but many were able to earn full marks on this part.

Q.6 As is usually the case, most candidates were able to pick up the majority of the marks on the area question.

Q.7 Some of the proofs in part (*a*) were unconvincing and in part (*b*), much of the logarithmic manipulation was poor, with only a minority getting the correct final answer of $\log_a 2$.

Q.8 It was only part (*b*)(i) which caused any real problems here. It seems clear that some candidates did not understand the meaning of 'touch' in this context and many thought that the fact that $AB <$ sum of the radii meant that the circles had to touch. Only a minority realised that C_1 actually lies inside C_2.

Q.9 Possibly due to the fact that the algebra involved this year was a little easier than what has been the case in previous years, many candidates got full marks on this question.

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Unit	Entry	Max Mark	Mean Mark
C3	1543	75	48.6

Grade Ranges

A	57
B	48
C	39
D	31
E	23

N.B. The marks given above are raw marks and not uniform marks.

C3

General Comments

This turned out to be a slightly more difficult paper that last year's. The majority of candidates found almost all questions to be accessible but the use of interval notation in questions 9 and 10 was sometimes poor.

Individual questions

Q.1 Part (*a*) caused very few problems. Many candidates, however, thought that the answer to part (*b*) was $\frac{1}{1{\cdot}6607} = 0{\cdot}6022$.

Q.2 This question was well answered although some candidates substituted $1 + \text{cosec}^2\theta$ for $\cot^2\theta$ whilst others thought that $\text{cosec}\,\theta = \frac{1}{\cos\theta}$.

Q.3 The only problems which arose here appeared in part (*b*)(ii). Many candidates did not realise that they were dealing with a quadratic equation whilst some thought that the fact that they could not see how to factorise this equation was sufficient to show that it had no real roots.

Q.4 Part (*a*) caused some problems but part (*b*) was generally well answered although some candidates lost the final mark for an incorrect statement.

Q.5 Very few problems arose here although not all candidates were able to earn the final mark in part (*d*).

Q.6 All parts were well answered.

Q.7 In part (*a*), not all candidates realised they had to choose *a* and *b* with one positive and one negative. A common error in part (*b*) was to show that $x = 5$ was a solution and then give the final answer as $x = \pm 5$.

Q.8 The standard of some candidates' graphs was poor and asymptotic behaviour was often unclear. Many failed to indicate that the second graph eventually became less steep than the first as *x* increased.

Q.9 This question was generally well answered although in part (*b*), some candidates gave their answer as a closed rather than an open interval.

Q.10 As in question 9, the main problem here seemed to be one of notation. It was not uncommon to see e.g. $R(g) = (-2, -\infty)$ or $R(g) = (-\infty, -2]$ as a solution in part (*a*). Most candidates who got to the end of part (*c*) did realise they had to reject $x = 4$ as a possible solution since it lay outside the domain of *fg*.

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Unit	Entry	Max Mark	Mean Mark
C4	2746	75	44.4

Grade Ranges

A	56
B	49
C	42
D	35
E	28

N.B. The marks given above are raw marks and not uniform marks.

C4

General Comments

Candidates found this to be an accessible paper but probably slightly more difficult than last year's paper. The questions which caused most problems were questions 5 and 9(*b*), and in particular, question 3(*a*).

Individual questions

Q.1 This question was well answered with many candidates getting full marks.

Q.2 This was another question which caused very few problems.

Q.3 Only a small number of candidates managed to get full marks in part (*a*). Many did not know the correct formula for tan 2x and did not realise that it was deducible from the information given in the Formula Booklet. Algebraic manipulation was often poor and many candidates lost two roots by dividing throughout by tan x. Finally, several candidates who had got that far failed to solve $2\tan^2 x - 1 = 0$ correctly. On the other hand, part (*b*) was well answered.

Q.4 Most candidates were able to earn the majority of the marks on this question.

Q.5 The fact that y^2 actually appeared in an expression given in the question seemed to cause some confusion. Some candidates tried to take the square root of $9 - x^2$ and then square their answer, but unfortunately made algebraic errors in doing so. Only a minority were able to say that their volume represented that of a sphere of radius 3.

Q.6 Overall, this question was well answered, but not all candidates were able to write down the range of values of x for which the expansion was valid.

Q.7 Neither part caused any general problems although some candidates got their limits the wrong way round in part (*b*).

Q.8 In part (*b*), most candidates were able to carry out the integration correctly, but not all were then able to get the expression for N in the required form. In part (*c*)(i), some candidates tried to find the value of k by rewriting their equations as logarithmic equations but unfortunately sometimes made algebraic errors in doing so. Many candidates, however, were able to earn full marks in part (*c*)(ii).

Q.9 Part (*a*) was well answered. In part (*b*), many candidates wrote down

$$\mathbf{r} = 8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \lambda\{(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) - (8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k})\}$$

rather than $\mathbf{r} = 8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$,

whilst others omitted the **r** on the left hand side. The majority of candidates, however, did realise that in order to show that L_1 and L_2 did not intersect, it was necessary to show that the equations derived from the expressions for the coefficients of each of **i**, **j** and **k** were inconsistent.

Q.10 Many candidates got full marks on this question and performance was generally much better than it has been on some of the questions involving proof by contradiction which have appeared in recent years.

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Unit	Entry	Max Mark	Mean Mark
FP1	235	75	41.2

Grade Ranges

A	54
B	47
C	40
D	33
E	26

N.B. The marks given above are raw marks and not uniform marks.

FP1

General Comments

The overall standard was disappointing with some candidates not suited to an examination at this level. On the other hand, some excellent scripts were seen. The manipulative work was often careless and the presentation of proof by mathematical induction continues to be generally poor.

Individual Questions

Q.1 This question was well answered by most candidates. It was noted, however, that some candidates expanded $(x+h)^3$ by first expanding $(x+h)^2$ and then multiplying by a further $(x+h)$. Candidates at this level should be familiar with Pascal's triangle and therefore able to expand $(x+h)^3$ immediately.

Q.2 Solutions to this question were generally good, the most common error being a failure to factorise the final answer.

Q.3 This question was well answered in general although some candidates seemed unfamiliar with the term complex conjugate.

Q.4 Most candidates were able to expand the determinant in (a) and show that the matrix was singular. In (b), most candidates knew what had to be done but the layout was often poor. Candidates should be encouraged to carry out the row reduction in a systematic way.

Q.5 Solutions to this question were often disappointing. Most candidates stated, correctly, that 2 – i was a root but were often unable to find the other roots. The most successful method was to find the other quadratic factor and hence the double root. Another successful method was to show that the other two roots, α and β, satisfied the equations $\alpha + \beta = -2, \alpha\beta = 1$ and then to find the values of α and β, often by inspection. Candidates who used the factor theorem to show that –1 was a root were given partial credit, not full credit because that method did not indicate that there were no other roots.

Q.6 As reported in previous years, the presentation was often extremely poor – indeed attempts at solutions using mathematical induction continue to be generally below what can reasonably be expected for candidates working for a qualification in Further Mathematics. Having established that the result is true for $n = 1$, the proof should start with a statement such as 'Assume that the result is true for $n = k$'. Instead of this, many candidates just write 'Let $n = k$' or even '$n = k$'. The next line should then state something like 'Consider, for $n = k + 1$' followed by the appropriate algebra. Many candidates just write 'Let $n = k + 1$'. Candidates should then round off the proof with something along the lines of 'Assuming the result to be true for $n = k$ implies that the result is true for $n = k + 1$ and since we have shown it to be true for $n = 1$, the general result follows by induction'. Many candidates finish with an incorrect statement such as 'Therefore true for k and $k + 1$ so proved by induction'.

Q.7 Most candidates were able to obtain correctly the matrix representing *T*. Solutions to (b), however, were often disappointing. The most successful method for this type of problem is to define the given line parametrically, here $x = \lambda, y = 2\lambda - 1$, to find the corresponding image under *T*, and then finally to eliminate λ.

Q.8 Candidates were generally successful in (a). However, many candidates failed to follow the instruction in (b) to use the result in (a) to derive the required result. Candidates were expected to multiply the result in (a) by **A** and then to substitute for $\mathbf{A}^2$ using the result in (a) again. Candidates who used other methods to solve (b) were given no credit.

Q.9 Most candidates began by assuming, correctly, that the roots can be expressed as a, ar, ar^2 and then wrote down the expressions involving sums and products of the roots. Candidates who realised that $a^2r + a^2r^2 + a^2r^3 = ar(a + ar + ar^2)$ then went on to solve the problem. Some candidates, however, seeing the result to be proved, expanded $(a + ar + ar^2)^3$ to give 27 terms which usually resulted in algebraic errors being made.

Q.10 Solutions to this question were often disappointing and it was the worst answered question on the paper. Some candidates failed to obtain correct expressions for *u* and *v* and were therefore unable to proceed to (b). Many of the candidates who obtained correct expressions for *u* and *v* were unable to carry out the necessary substitution and elimination required in (b).

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Unit	Entry	Max Mark	Mean Mark
FP2	315	75	44.7

Grade Ranges

A	52
B	44
C	37
D	30
E	23

N.B. The marks given above are raw marks and not uniform marks.

Paper FP2

General Comments

The standard of the scripts was generally good.

Individual Questions

Q.1 Most candidates solved this question correctly. Some candidates, however, evaluated the final answer using their calculator in degree mode which gave 10.17. Candidates at this level should realise that this is an impossible answer to the given integral.

Q.2 Solutions to this question were generally good. Many candidates obtained correct values of the cosine of multiples of θ but then, in trying to find the general solution, added the $2\pi n$ too late, eg instead of stating that

$$\cos 2\theta = -\frac{1}{2} \Rightarrow 2\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$$

they wrote, incorrectly, that

$$\cos 2\theta = -\frac{1}{2} \Rightarrow 2\theta = \pm\frac{2\pi}{3} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$$

Q.3 Most candidates solved (a) correctly by noting that there was a sudden jump passing through $x = 2$. Solutions to (b), however, were often incomplete. Although most candidates obtained correctly the derivatives of $f(x)$ in the two regions, some then showed simply that the values of these derivatives at $x = 2$ were positive and thought that was enough to prove that f is strictly increasing. Many candidates showed that the derivatives were positive for all x and thought that was enough to prove that f is strictly increasing. Only a few candidates realised that it was also necessary to mention that the jump at $x = 2$ is positive – clearly a negative jump would indicate that f is not strictly increasing. Many candidates gave [–2,7] as their answer to (c), completely forgetting the jump at $x = 2$. It was surprising that only a handful of candidates sketched the graph of f – this would have made the function so much easier to deal with.

Q.4 Parts (a) and (b) were well done in general with most candidates obtaining correct values for the modulus and argument of z. It was also pleasing to note that many candidates calculated the three cube roots correctly. Part (c), however, defeated many candidates and $n = 4$ instead of $n = 8$ was often seen.

Q.5 Part (a) was well answered by most candidates and many went on to solve (b) correctly. The most common errors, not often seen, were incorrect signs in the expansion of $\left(z - \frac{1}{z}\right)^5$ and incorrect or no combinatorial terms in the expansion.

Q.6 Solutions to this question were generally disappointing. Some candidates were unable to complete the square correctly which meant that they could not locate the centre of the ellipse. Some of the candidates who completed the square correctly were unable to identify the lengths of the major and minor axes so that the calculation of eccentricity could not be carried out successfully. Many candidates forgot that the foci and directrices had to be translated to allow for the fact that the ellipse was not centred at the origin.

Q.7 Most candidates were unable to answer this question, seemingly unaware of the result that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_a^x f(t)\mathrm{d}t\right) = f(x)$$

Some candidates tried to integrate $\sin(\mathrm{e}^t)$, sometimes taking a page or more, without success of course because $\sin(\mathrm{e}^t)$ does not have an indefinite integral. Candidates should be aware that if part of a question is allocated just 1 mark, then the answer can be obtained in no more than a few lines.

Q.8 Solutions to (a) were often disappointing. Some candidates tried to express *f*(*x*) in partial fractions which was invalid because the degrees of the numerator and denominator were equal. This was, however, given partial credit because this method gives the second and third terms but not, of course, the first. It was disappointing to note in (b) that candidates who stated that the stationary points satisfied

$$\frac{4}{(x-1)^2} = \frac{9}{(x-2)^2}$$

almost always cross multiplied to give a quadratic equation instead of the quicker method of taking square roots to give

$$\frac{2}{(x-1)} = \pm\frac{3}{(x-2)}$$

In (d), the graph was often drawn incorrectly and the middle branch was sometimes omitted.

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Unit	Entry	Max Mark	Mean Mark
FP3	177	75	45.9

Grade Ranges

A	50
B	44
C	38
D	32
E	26

N.B. The marks given above are raw marks and not uniform marks.

FP3

General Comments

The number of candidates was almost double the number who sat FP3 in 2010 and although there were some excellent candidates this year, it was felt that the overall standard of the candidates had dropped slightly.

Individual Questions

Q.1 This was well answered by most candidates. The most common error, not often seen, was to substitute $\text{sech}^2 x - 1$ for $\tanh^2 x$ instead of $1 - \text{sech}^2 x$. A useful rule here is Osbourne's Rule which states that in going from a trigonometric identity to the corresponding hyperbolic identity, a sign change is necessary for terms which are products, or implied products, of sines.

Q.2 Somewhat unexpectedly, this was the worst answered question on the paper. It was surprising to note that many candidates were unaware that when putting $t = \tan\frac{x}{2}$, d*x* should be replaced by $\frac{2\text{d}t}{1+t^2}$; some candidates failed to go beyond $\frac{2\text{d}t}{\sec^2\frac{x}{2}}$, not knowing how to eliminate *x*. Many candidates failed to realise that completing the square should be used to simplify $t^2 + t + 1$ and incorrect integrals involving natural logs were sometimes given.

Q.3 This was one of the better answered questions on the paper. Although the integration involved manipulation of hyperbolic functions including double angles, many candidates completed the integration successfully.

Q.4 Parts (a) and (b) were well answered by most candidates. In (c), however, some candidates failed to realise that in addition to differentiation, substitution of the series from (b) was required to obtain the printed result.

Q.5 Most candidates were able to derive the Newton-Raphson formula and to use it successfully to solve the equation. In (c), however, many candidates were unable to differentiate *f*(*x*) correctly, the most common result being

$$\frac{1}{\sqrt{1-\left(\frac{0.5}{x}\right)^2}}$$

with candidates not realising that the chain rule was required to produce an extra factor. The most successful method here was to write

$$y = \sin^{-1}\left(\frac{0.5}{x}\right) \Rightarrow \sin y = \frac{0.5}{x} \Rightarrow \cos y\frac{\text{d}y}{\text{d}x} = -\frac{0.5}{x^2} \Rightarrow \frac{\text{d}y}{\text{d}x} = -\frac{0.5/x^2}{\sqrt{1-(0.5/x)^2}}$$

Q.6 This was well answered in general. Most candidates solved (a) correctly but the algebra in (b) defeated some candidates.

Q.7 Candidates who split the integrand into $\tanh^{n-2} x \tanh^2 x$ were usually successful in deriving the reduction formula. Some candidates, however, split the integrand into $\tanh^{n-1} x \tanh x$ and then tried to use integration by parts but this was invariably unsuccessful. Many candidates, even those who were unable to obtain the reduction formula, solved (b) correctly. It is important for candidates to realise that even if they cannot solve the first part of a question, it is sometimes possible to solve later parts of that question.

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