



# **GCE EXAMINERS' REPORTS**

**MATHEMATICS  
AS/Advanced**

**SUMMER 2009**

## **Statistical Information**

This booklet contains summary details for each unit: number entered; maximum mark available; mean mark achieved; grade ranges. *N.B. These refer to 'raw marks' used in the initial assessment, rather than to the uniform marks reported when results are issued.*

## **Annual Statistical Report**

The annual *Statistical Report* (issued in the second half of the Autumn Term) gives overall outcomes of all examinations administered by WJEC.

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# MATHEMATICS

## General Certificate of Education 2009

### Advanced Subsidiary/Advanced

*Principal Examiner:* Dr E Read

#### Unit Statistics

The following statistics include all candidates entered for the unit, whether or not they 'cashed in' for an award. The attention of centres is drawn to the fact that the statistics listed should be viewed strictly within the context of this unit and that differences will undoubtedly occur between one year and the next and also between subjects in the same year.

<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
C1	2926	75	45.0

#### Grade Ranges

A	58
B	50
C	42
D	34
E	27

*N.B. The marks given above are raw marks and not uniform marks.*

## C1

### General Comments

In general, performance on this paper seemed to be on a par with that of previous summers but marks are still being lost due to errors being made in simple arithmetic computation and algebraic manipulation. It was only the last two marks in question 10 which caused general problems.

### Individual questions

- Q.1 As usual, this was a well answered question. Relatively few candidates, however, managed to gain the final two marks.
- Q.2 (a) Very few problems.
- (b) Not all candidates were able to simplify  $\frac{14}{\sqrt{2}}$ .
- Q.3 Almost all candidates now realise that differentiation must be the starting point for this type of question. Some, however, used the negative reciprocal of  $\frac{dy}{dx}$  as the gradient of the tangent.
- Q.4 (a) The fact that  $a$  was not a whole number meant that many candidates made errors of computation when trying to find the value of  $b$ . Relatively few realised that the required greatest value was  $-b$ .
- (b) Only a minority of candidates were able to gain the final mark by writing down a geometrical interpretation of their result.
- Q.5 (a) In the proof from first principles, there continues to be some evidence of improved use of correct mathematical notation.
- (b) Seemed to cause few problems.
- Q.6 Although the majority of candidates were able to write down an expression for the discriminant in terms of  $k$ , only relatively few were able to show that this expression was always equal to 4.
- (b) Was well done, as is usually the case.
- Q.7 (a) Most candidates were able to expand the given expression but many were unable to simplify their expansion correctly.
- (b) There were many correct solutions, although some candidates wrote down equations containing  $x$  rather than just involving the coefficients.
- Q.8 Questions involving the Remainder and Factor theorems are always well answered and this was no exception.
- Q.9 Candidates seemed to find both parts of this question relatively straightforward. Most of the errors that were made seemed to be occur in part (b).
- Q.10 Part (a) caused hardly any problems at all. However, many candidates did not seem to know how to verify that the stationary point was a point of inflection. Although there were some correct solutions, a large number of candidates seemed to believe that  $\frac{d^2y}{dx^2} = 0$  at the stationary point was a sufficient condition.

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
C2	3868	75	42.7

#### Grade Ranges

A	59
B	50
C	42
D	34
E	26

*N.B. The marks given above are raw marks and not uniform marks.*

## C2

### General Comments

Candidates found this to be an accessible paper and overall performance was good. However, some candidates still lack confidence in carrying out simple algebraic manipulation. There was also some evidence of poor use of the calculator (premature approximation) and some of the standard basic proofs are still not fully understood.

### Individual questions

- Q.1 Most seemed to find this quite a straightforward question. It is important, however, that candidates work to sufficient decimal places to ensure that their answers have the required degree of accuracy.
- Q.2 (a) Was well answered.
- (b) It was disappointing that some candidates who correctly found the possible values for  $2x + 12^\circ$  were then unable to find the corresponding values for  $x$ . Others started by assuming that  $\sin(2x + 12^\circ) = \sin 2x + \sin 12^\circ$ .
- Q.3 Although many candidates were able to gain full marks on this question, there were others who seemed to be unfamiliar with the concept of the ambiguous case which can arise in the construction of triangles.
- (b) (i) Not all candidates appreciated that all they needed to use was the fact that the angle sum of a triangle is  $180^\circ$ .
- Q.4 (a) Some candidates did not write down sufficient detail when proving the formula for the sum of the first  $n$  terms of an arithmetic progression.
- (b) Most seemed to find part (b) relatively straightforward,
- (c) Relatively few candidates were able to correctly simplify their expression to give either  $n(2n + 1)$  or  $2n^2 + n$ .
- Q.5. (a) most candidates were able to conclude that  $r = 3$ . Many then calculated the value of  $a$  and used the fact that  $t_7 = ar^6$ . Not all got the correct answer. All that was really needed was to divide 36 by  $3^2$ .
- (b) The majority of candidates seemed to be able to set up and solve their equations correctly.
- Q.6 (a) Not all candidates remembered to include the constant of integration.
- (b) Was generally well answered.
- Q.7 (a) Whilst it is clear that many candidates are able to reproduce the type of logarithmic proof asked for, there are always others who seem to go round in circles.
- (b) Some lost the final mark by approximating prematurely.
- (c) Seemed to be better answered than similar questions which have been set in recent years.

- Q.8 (a) Caused few problems.
- (b) Most candidates seemed to know how to find the equation of the tangent.
- (c) Many candidates realised how to work out what was in effect the least distance between the circles.
- Q.9 (a) Candidates found this relatively straightforward but many seemed to think that their value for  $\theta$  could in some way be used to find  $\phi$  in part (b).

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### Advanced Subsidiary/Advanced

Chief Examiner: Dr R H Thomas

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Unit	Entry	Max Mark	Mean Mark
C3	1364	75	45.6

#### Grade Ranges

A	55
B	46
C	37
D	28
E	20

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### C3

There was deterioration in candidate performance compared with previous years, the mean examination mark decreasing by about four points.

The deterioration appeared to be mainly due to the candidates' failure to cope with some questions which explored more than one theme. Thus, for example, whilst most candidates are familiar with parametric differentiation and differentiation of a quotient, question 3 which involved both topics proved difficult for many.

In question 4, candidates were required to derive an equation (by considering the stationary value of a function) before solving this equation by iteration. Whilst both requirements were within the capacity of many candidates, their simultaneous presence in question 4 resulted in loss of marks.

Similar remarks apply to question 9 and will be highlighted later.

As in previous years, lack of algebraic skills was apparent in many answers.

More detailed comments are given below:

Q.1 This question provided most candidates with a good introduction to the examination. It should be noted that if an answer correct to four decimal places is required then intermediate calculations should involve at least five (preferably more) decimal places. Candidates are reminded that writing down an answer with no working gains no marks.

Q.2 (a) Generally well done.

(b) A high scoring question for most candidates although the following points arose frequently:

i.  $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$ , so that  $\operatorname{cosec} \theta = 1 + \cot \theta$

ii.  $\operatorname{cosec} \theta = \frac{1}{\cos \theta}$

Q.3 (a) Generally well done.

(b) There were a high number of good answers although, as mentioned earlier, the use of differentiation of a quotient in a question involving parameter differentiation caused some difficulty. It was disappointing at C3 level to observe  $(3 + 2t)^2$  expanded as  $9 + 4t^2$ . As it happens, candidates were not required to provide that expansion.

Q.4 (a) The derivation of the equation in  $x$  was beyond the capabilities of most candidates, apparently. Common errors occurring related to incorrect use of the product rule and/or incorrect differentiation of  $e^{2x}$ .

(b) Generally well answered. Candidates are reminded that a clear statement about the change of sign  $(x - 1)e^{2x} - 1$  leads to the conclusion that there is a root between  $x = 1$  and  $x = 2$ , is necessary. Also the result of final analysis is that  $\alpha$  (or the root) is 1.1089. Many candidates stated that the final result is that  $x_3 = 1.1089$ , correct to four decimal places.

Q.5 Generally well answered.

Q.6 (a) Candidates should state the solution is  $x \leq \frac{10}{9}$  and  $x \geq \frac{4}{9}$  or more succinctly  $\frac{4}{9} \leq x \leq \frac{10}{9}$ .

(b) Surprisingly, relatively few candidates were able to gain the 2 marks available for this question. A direct approach involving

$$5|x| + 1 = 9$$

$$5|x| = 8$$

$$x = \pm \frac{8}{5}$$

would have given a quick reward. However, some candidates considered the cases

$$\sqrt{5x + 1} = 9 \quad \text{for } x > 0,$$

$$\text{and } \sqrt{-5x + 1} = 9 \quad \text{for } x < 0,$$

This correct first-principles approach unfortunately resulted in disastrous consequences.

Q.7 Generally well answered, although it should be noted that the indefinite integrals in (a) (i) & (ii) required the addition of arbitrary constants in the answers.

Q.8 Transformations continues to be a source of difficulty for many candidates, in spite of the topic being part of the GCSE and C1 specifications. The difficulty appears to arise when combinations of transformations are involved. The major difficulty was associated with the scaling in the  $y$ - direction.

Q.9 Candidates appeared to possess knowledge of composition of functions, appreciating for instance, that the formulation of  $fg$ ,  $g$  was applied first. The major difficulty was the simplification of  $3e^{2\ln 4x}$ , by using the properties of logs and exponentials. As in previous years, candidates did not appreciate the need to check the domain of a function in order to discriminate between possible solutions of equations. For instance, one approach in (c) yielded the equation  $48x^2 = 12$ . Consideration of the domain led to the rejection of the apparent root  $x = -\frac{1}{2}$ .

Q.10 (a) Whilst most candidates were able to obtain  $f'(x) = \frac{12x}{(3x^2 + 2)^2}$ , they were less successful in explaining why  $f'(x) > 0$ .

(b) The upper point of the range posed difficulty for many candidates who failed to realise that as  $x \rightarrow \infty$ ,  $f(x) \rightarrow 1$ .

(c) As in previous years, many candidates' inadequacy in changing the subject of a formula was revealed: much of the algebraic manipulation was disappointing.

There was also a general failure in discriminating between the possible values of  $f^{-1}(x)$ . Reference to the domain of  $f$  would have revealed that the positive sign was the appropriate choice.

# MATHEMATICS

## General Certificate of Education 2009

### Advanced Subsidiary/Advanced

*Principal Examiner:* Dr R H Thomas

#### Unit Statistics

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
C4	2371	75	43.5

#### Grade Ranges

A	55
B	48
C	41
D	34
E	28

*N.B. The marks given above are raw marks and not uniform marks.*

## C4

As with C3, the candidates' response to the paper appeared less positive than in previous years. That notwithstanding, the general performance was comparable with those of previous years.

As in C3, candidates were unable to deal satisfactorily with questions requiring knowledge of more than one technique or topic. Thus, for example, there were few solutions of Q.5 that were completely correct.

Algebraic skills were often inadequate. These deficiencies were evident in questions 5, 7, 9 and 10.

Q.1 A high scoring question for most candidates. Marks were lost mainly as a result of faulty integration of  $\frac{-3}{(1+x)^2}$  and  $\frac{-6}{2+x}$ .

Q.2 Surprisingly, few candidates were able to gain full marks. The main sources of difficulty were:

- i. Use of incorrect expansion formula for  $\sin 2\theta$ ,  
 $\sin 2\theta = \cos^2 \theta - \sin^2 \theta$  being a popular suggestion.
- ii. The neglect of the possibility that  $\sin \theta = 0$ .

Q.3 Generally well answered. However, very few students realised that if  $\cos(\theta - 60^\circ) = \frac{1}{2}$  then one possibility is that  $\theta - 60^\circ = -60^\circ$ .

Q.4 A very low scoring question, few students being able to evaluate  $\int_0^{\frac{\pi}{8}} \cos^2 2x \, dx$ .

Apparently, nearly all candidates could have evaluated  $\int_0^{\frac{\pi}{8}} \cos^2 x \, dx$ , indeed many changed the question in order that they could demonstrate their knowledge of that standard case. It was disappointing to observe that there was little appreciation of the general rule, whereby the integral of  $\cos^2(\ )$  could be found by considering the integral of  $\cos 2(\ )$ .

Q.5 (a) Generally well answered.

(b) All but the weaker candidates were able to derive the equation  $q^3 - 3q^2 + 4 = 0$ . In contrast, most attempts at solving the equation were disappointing. Few candidates recalled the factor theorem. It was disturbing to observe the common approach in solving the equation:

$$q^2 (q - 3) = 4$$

so that

$$q^2 = -4 \quad \text{or} \quad q - 3 = -4.$$

- Q.6 (a) Generally, integration by parts was not as well worked as previous years, mainly due to incorrect choices of  $u$  and  $\frac{dv}{dx}$  in

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx.$$

- (b) Whilst a number of candidates were able to make progress with the integration by substitution, the general improvement in this area continues.
- Q.7 (a) Except for the weaker candidates (most of them wished to change the question to one involving experimental decay); many were able to make good progress. However, relatively few candidates were able to gain full marks; because they were unable to relate convincingly the proportionality constant (usually  $k$ ) to the constant  $A$  appearing in the final result.

- (b) Generally well answered.

- Q.8 There were many good attempts at this high scoring question.

- (a) Candidates are reminded that the vector equation of a line is
- $$\mathbf{r} = ( ) \mathbf{i} + ( ) \mathbf{j} + ( ) \mathbf{k}$$

or

$$\mathbf{OP} = ( ) \mathbf{i} + ( ) \mathbf{j} + ( ) \mathbf{k},$$

where  $O$  is the origin and  $P$  is a general point on the line; and

$$\text{equation} = ( ) \mathbf{i} + ( ) \mathbf{j} + ( ) \mathbf{k}$$

is not an appropriate answer.

- (b) Generally well answered.

- (c) Generally well answered although there was some abuse of notation, candidates did not appear to appreciate that the scalar product gives rise to a scalar.

- Q.9 The expansion of  $(1+4x)^{\frac{1}{2}}$  was derived satisfactorily by most candidates, although a substantial number were unable to state the restriction on  $x$  required to ensure convergence. Only a few candidates realised that the second expansion could be obtained by an appropriate substitution for  $x$ .

- Q.10 There was a generally disappointing response, only the most able candidates provided convincing arguments. There was a failure to derive

$$3b^2 = 9k^2$$

$$b^2 = 3k^2$$

and to state that  $b^2$  (and therefore  $b$ ) possessed the factor 3.

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
M1	2153	75	45.1

#### Grade Ranges

A	56
B	47
C	39
D	31
E	23

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## M1

### General Comments

This paper was comparable in standard and length to recent past papers on this syllabus and presented little difficulty to candidates in general. All the questions were accessible and there was not a question which proved to be difficult generally. There were a number of candidates who obtained full marks.

### Comments on individual question

- Q.1 Sign errors were very common throughout the question. Candidates were often uncertain whether they were using upwards positive or downwards positive. Disappointingly, many candidates who obtained the correct equation in part (c) failed to recognise it as a quadratic equation and consequently lost the last 2 marks.
- Q.2 Part (a) of this question was well done generally. Only a small number of candidates were awarded the mark in part (b).
- Q.3 A small number of candidates missed out the mass for the person in the lift in part (a). Sign errors were common in part (b), especially for those who obtained negative acceleration in (a).
- Q.4 Many candidates correctly tried to use the equation  $v = u + at$  but unfortunately used  $t = 10$  instead of the correct  $t = 30$ . This led to the answer  $v = 35$  in part (b). As the  $v$ - $t$  graph has a maximum  $v$  value of 20, this should have, but did not, alert the candidates to the mistake. Part (c) was well done generally with some numerical errors due to the region from  $t = 0$  to  $t = 30$  being a trapezium rather than a triangle. In part (a), more than a handful of candidates did not realise that a flat line in a  $v$ - $t$  graph indicated constant velocity and therefore acceleration is zero.
- Q.5 Most candidates were able to accurately calculate the value of the limiting friction though a few sin/cos errors were seen. The most frequently occurring error is in the direction of the friction which should be acting up the plane to prevent motion downwards. However, many candidates thought the direction of friction opposed that of the applied force. The most serious error was made by candidates who thought that because there was no motion, there was also no friction acting. Part (b) was better done than part (a) as friction was acting to oppose the force acting on the object. The usual errors such as omitting the component of weight down the slope were common.
- Q.6 This question did not bring up any surprises. It was well done generally with the usual sign errors with the velocities in both parts of the question.
- Q.7 In part (a), most candidates were able to find the reactions at the pivot easily though candidates who took moments about two different points were usually unsuccessful. In part (b), candidates did this question in a variety of ways all involving moments. Some candidates introduced their own  $x$  into the solution and failed to convert it to the length required in the question.
- Q.8 In this question, sign errors and sin/cos errors as well as errors with calculating the complement of angles were widespread. Candidates obtained the two resolved components of the resultant easily but some did not manage to combine the two components by squaring and adding. Often, the two perpendicular components were simply added together.
- Q.9 Candidates had difficulty finding the height of the triangle and consequently the area and the centre of mass of the triangle were incorrectly calculated. Some candidates failed to spot the symmetry in the lamina.

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M2	751	75	49.5

### Grade Ranges

A	58
B	50
C	42
D	34
E	27

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## M2

### General Comments

There were no surprises on this paper. The standard of the questions and the length of the paper were appropriate to the syllabus and the level of competence of the candidates. Many candidates obtained full marks.

### Comments on individual questions

- Q.1 Sign errors in integrating sine and cosine functions were very common and it was disappointing to see so many candidates failing to calculate  $\sin(2\pi)$  and  $\cos(\pi)$ ,  $\sin(\pi/2)$  and  $\cos(\pi/4)$  correctly; many candidates had their calculators set to degrees. In part (b), many candidates assumed that the constant of integration was 4 without substituting the initial conditions into the equation obtained by integration.
- Q.2 In part (a), many candidates used  $x = -0.05$  because it is a compression but failed to use  $T = -80$  because it is a thrust, thereby obtaining a negative modulus of elasticity without being alerted to the mistake. Many others used  $x = 0.2$ , possibly because of misreading the question. In part (b), the kinetic and the elastic energies were usually correctly calculated. However, many candidates included a term for potential energy in spite of the fact that motion is horizontal.
- Q.3 The question asked for work done against friction; the correct formula is therefore friction  $\times$  distance. Many candidates used the formula force  $\times$  distance with the force not being the friction. The main error in part (b) is the omission of the work done calculated in part (a) in the Work-Energy equation.
- Q.4 This question was well done generally. Many candidates obtained full marks. The common mistake is the omission of the resistance in part (a) and the inclusion of resistance in part (b). Some candidates did not realise that the acceleration is zero when the speed is constant.
- Q.5 This question was generally very well done indeed. A small number of candidates interchanged initial horizontal and vertical velocities which was either a sin/cos error or misreading the tangent given in the question.
- Q.6 In part (a), many candidates did not realise momentum could be a vector quantity and attempted to change a perfectly correct answer to a scalar by a variety of incorrect methods. Many candidates who were not awarded any marks in (a) manage to recover in part (b). Candidates who obtained a  $t$ -dependent expression for the acceleration nevertheless went on to state that it is constant. Part (c) was well done generally though many candidates made algebraic errors in the scalar product and in the solution of a perfectly simple linear equation which resulted from the scalar product.
- Q.7 This topic (horizontal circular motion) does not seem to be well known by many candidates. The fact that there is acceleration towards the centre of the motion is well known, as is the formula for it. However, many candidates considered the problem as a static one, ignoring the acceleration, instead of using Newton's second law in two directions. Some candidates obtained two perfectly correct equations but were unable to proceed towards a correct solution.
- Q.8 Almost everyone considered energies in part (a). The most common error is some mistake with the potential energy. Part (b) was not as well done as (a) as many omitted the component of weight of the particle when considering N2L towards the centre of motion. There were also sign and algebraic errors when simplifying the expression. In part (c), the most common error is the consideration of the sign of  $v^2$  at the top of motion rather than the sign of the reaction obtained in (b).

# MATHEMATICS

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*Principal Examiner:* Dr S Barham

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
M3	137	75	57.1

#### Grade Ranges

A	59
B	51
C	43
D	35
E	27

*N.B. The marks given above are raw marks and not uniform marks.*

## M3

### General Comments

This was a good paper which did not present any difficulties to most candidates. There were numerous excellent scripts and there was a high average mark from the cohort. It was a pleasure to mark this paper.

### Comments on individual questions

- Q.1 This question was generally very well done, with a handful of candidates forgetting the constant of integration and making a mistake with the coefficient or the sign of the  $\ln$  term when integrating.
- Q.2 The equations for Simple Harmonic Motion are generally well known and candidates were able to apply them appropriately. There were a few errors with the amplitude and the period.
- Q.3 As in question 1, this question was generally well done with all the same problems when integrating for a minority of candidates.
- Q.4 Most candidates were able to solve this question correctly. Some candidates using the conservation of momentum (which is correct here as motion is all in one direction) in part (b) forgot to add the masses of the objects and obtained the wrong velocity. Very few candidates were not able to do part (c).
- Q.5 A few candidates were not able to write down the complementary function from complex roots of the auxiliary equation. Some attempted to find the particular solution before finding the particular integral and hence the general solution.
- Q.6 This question was much better answered than the ones on this topic in past papers and many candidates obtained full marks. Common mistakes were sin/cos errors and errors in algebraic manipulation. Candidates who found the numerical value of the force at  $B$  and then substituted to obtain numerical values for the reaction at the floor and the friction on the floor before attempting to find the minimum value of the coefficient of friction were generally more successful.

# MATHEMATICS

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*Principal Examiner:* Dr J Reynolds

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
S1	2371	75	42.4

#### Grade Ranges

A	52
B	45
C	38
D	31
E	25

*N.B. The marks given above are raw marks and not uniform marks.*

## General Comments

The standard of the scripts was somewhat variable with some excellent work. On the other hand, a significant number of candidates were clearly unprepared for an examination at this level. The general response to questions on continuous distributions continues to be disappointing due to lack of confidence dealing with integration.

## Comments on Individual Questions

- Q.1 This question was well answered by the majority of candidates. As noted in earlier reports, methods involving multiplication of probabilities and the use of combinatorics were seen in approximately equal measure.
- Q.2 Part (a) was well answered by most candidates although a few answered (i) and (ii) the wrong way round. In (b), those candidates who drew a Venn diagram were generally more successful than those who relied on algebra alone. Few candidates answered (c) correctly with many candidates stating, incorrectly, that the smallest value of  $P(A \cup B)$  occurred when  $A$  and  $B$  were independent.
- Q.3 Parts (a) and (b)(i) were well answered in general although (b)(ii) caused problems for some candidates who obtained correct equations for  $a$  and  $b$  but were then unable to solve them correctly.
- Q.4 Part (a) was well answered in general although some candidates misread the tables. Part (b) proved to be difficult for many candidates who, although showing that  $e^{-0.6t} = 0.5$ , were then unable to solve this equation. Candidates were expected to realise that this equation was of the type  $a^x = b$  which can be solved using logarithms. Candidates who used tables to see that  $-0.6t \approx 0.7$  were given partial credit.
- Q.5 Straightforward questions on the use of the Law of Total Probability and Bayes' Theorem are usually well answered and this was no exception. Candidates who drew a tree diagram were generally more successful.
- Q.6 Part (a) was well answered in general although some candidates appear to believe that  $\text{Var}(X) = E(X^2)$ . In (b), some candidates misunderstood the question completely, believing that  $P(X_1 = X_2)$  had something to do with  $P(X_1) = P(X_2)$ .
- Q.7 Few candidates gave a correct solution to (a). Most candidates were unable to list the three situations in which Ann obtains more heads than Bob and many of those who did assigned incorrect probabilities. Many candidates solved (b) (i) and (ii) correctly but some failed to realise that the solution to (iii) required the summation of an infinite series.

Q.8 Solutions to this question were often disappointing with many candidates showing a poor understanding of integration. In (a), some candidates evaluated  $E(X)$  incorrectly as  $\int_0^1 f(x) dx$  which, not surprisingly, gave  $E(X) = 1$ . Solutions to (b) were also often

disappointing with  $F(x) = \int_0^1 f(x) dx = 1$  seen. The incorrect notation  $F(x) = \int_0^x f(x) dx$

was not uncommon. It is incorrect to use the same letter to denote both the upper limit and the variable of integration – this will only cause confusion to candidates studying mathematics to a higher level. The limits were often omitted – it is of course a valid method to state that

$F(x) = \int f(x) dx + C$  and then choose  $C$  so that either  $F(x) = 0$  at the lower limit or

$F(x) = 1$  at the upper limit although it is more advisable to find  $F(x)$  using a definite integral as above. In (c), it was disappointing to see so many candidates obtaining the correct quadratic equation  $m^2 + m = 1$  and then ‘solving’ this by stating that  $m(m+1) = 1$  so  $m = 1$  or  $m + 1 = 1$ .

# MATHEMATICS

## General Certificate of Education 2009

### Advanced Subsidiary/Advanced

*Principal Examiner:* Dr J Reynolds

#### Unit Statistics

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
S2	778	75	46.9

#### Grade Ranges

A	53
B	45
C	38
D	31
E	24

*N.B. The marks given above are raw marks and not uniform marks.*

## General Comments

The general standard was good with a handful of excellent scripts. Continuity corrections continue to be a source of difficulty for many candidates with either incorrect or no correction being used. In some cases, the interpretation of  $p$ -values is unsatisfactory – candidates are recommended to use the guidelines in the specification. Some candidates interpret a  $p$ -value as offering **some** evidence pointing to one or other of the hypotheses which is altogether too vague. Also, some candidates fail to give a conclusion in context when this is asked for.

## Comments on Individual Questions

- Q.1 This was well answered by most candidates. In (a), some candidates incorrectly defined the  $p$ -value as  $P(X = 18)$ . In (b), an incorrect or no continuity correction was sometimes seen. The required conclusion was that the small  $p$ -value indicated very strong evidence for an increase in mean.
- Q.2 Part (a) was well answered by most candidates, the most common error in (ii) being the incorrect assumption that the solution could be found by tripling the answer to (i). Part (b) was generally well answered.
- Q.3 Part (a) was generally well answered. In (b), candidates who used trial and error to deduce that the significance level was 95% were given no credit. Although the 95% confidence interval is [6.62, 6.74] correct to 2 decimal places, the actual confidence level of the exact interval given is 94.2% (or 94% to the nearest integer).
- Q.4 This question was well answered by most candidates, the most common error being to assume the standard error to be  $\sqrt{0.5^2 / 11}$ . Candidates who concluded that there is no difference between boys and girls were given no credit.
- Q.5 Part (a) was well answered by most candidates. In (b), however, although many candidates obtained the equation  $\mu = 9.36 - \mu^2$ , some candidates were unable to solve this equation. It remains an inexplicable fact that some candidates leave their Pure Mathematics at home when sitting a Statistics examination.
- Q.6 In (a), most candidates gave the correct expression for  $f(x)$  but many were unable to derive the correct expression for  $F(x)$ . In (b), many candidates wrote incorrectly that  $E(\sqrt{X}) = \sqrt{E(X)}$ . Part (b)(ii) caused problems for many candidates although some realised (perhaps fortuitously) that  $\text{Med}(\sqrt{X}) = \sqrt{\text{Med}(X)}$ .
- Q.7 In (b), candidates who used a normal approximation were given no credit as were candidates who failed to give their conclusion in context. Part (c) was generally well answered although some candidates were under the impression that different continuity corrections were required in (i) and (ii).

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
S3	112	75	52.0

#### Grade Ranges

A	55
B	48
C	41
D	34
E	27

*N.B. The marks given above are raw marks and not uniform marks.*

## General Comments

The standard of the scripts was generally good with some excellent scripts. Statistical inference is well understood and, unlike previous years, most candidates recognised the question on the Student's  $t$ -distribution. It was disappointing to find that so many of these Further Mathematics candidates were unable to deal successfully with the algebra required in Q.6.

## Comments on Individual Questions

- Q.1 While (a) was well done by almost all the candidates, only a minority recognised the binomial distribution in (b). Those candidates who found the probability distribution from first principles spent a lot of time earning a single mark.
- Q.2 This question was well answered by the majority of candidates.
- Q.3 Parts (a) and (b) were well answered in general. Solutions to (c), however, were often disappointing. Many candidates appear to believe that the Central Limit Theorem states that the distribution itself, rather than the sample mean, is approximately normal for large samples, apparently failing to realise that the taking of a sample cannot possibly affect the sampled distribution.
- Q.4 It has been reported previously that, to find the unbiased estimate of  $\sigma^2$ , some candidates divide by  $n$  instead of  $n - 1$  and then sometimes follow this up by multiplying by  $n/(n - 1)$ . It was pleasing to note that this method was rarely seen this time and most candidates used the direct formula

$$\frac{\sum x^2}{n-1} - \frac{(\sum x)^2}{n(n-1)}$$

It was also pleasing to see that most candidates realised that, since the sample was small and the variance was estimated, the Student's  $t$ -distribution should be used to determine the confidence interval.

- Q.5 This question was well answered in general.
- Q.6 Solutions to this question were generally disappointing. Most candidates showed that  $U$  was an unbiased estimator and many found the correct expression for the variance of  $U$ . Only a minority of candidates then realised that the best estimator corresponded to minimum variance and that differentiation was required although one script was seen in which the result was obtained by completing the square.

Most candidates who differentiated then showed correctly that  $\frac{\lambda}{1-\lambda} = \frac{m\sigma_y^2}{n\sigma_x^2}$

It was disappointing to note that some candidates were unable to carry on correctly to obtain an expression for  $\lambda$ , with their attempts at cross multiplication ending in error. No candidate seemed aware of the result (with the slightly strange name of

'componendo') that  $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{a+b} = \frac{c}{c+d}$  which gives the required expression for  $\lambda$  immediately.

- Q.7. Candidates are generally well prepared for questions on this topic and this was no exception. Most candidates were able to estimate  $\alpha$  and  $\beta$  successfully, usually by first calculating  $S_{xx}$  and  $S_{xy}$ . Part (b) was also well answered in general although only a minority of candidates were able to explain why  $\beta = 0.52$  was obviously incorrect.

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
FP1	210	75	46.3

#### Grade Ranges

A	59
B	51
C	43
D	36
E	29

*N.B. The marks given above are raw marks and not uniform marks.*

## FP1

### General Comments

Although some excellent scripts were seen, the overall standard was disappointing with a sizeable minority clearly not suited to an examination at this level. The manipulative work was often careless and the presentation of proof by mathematical induction continues to be generally poor.

### Comments on Individual Questions

- Q.1 This question was well answered by most candidates. The most common error was to multiply out some of the terms which made factorisation difficult later on.
- Q.2 Solutions to this question were often disappointing with some candidates apparently confused by the transition from a quadratic to a cubic. Some candidates decided to find the (complex) roots of the quadratic equation first although only one script was seen in which this led to a correct solution.
- Q.3 (a) It was pleasing to note, in contrast to previous years, that most candidates managed to find the inverse of the matrix without making algebraic errors along the way.
- (b) Some candidates failed to take note of the word 'hence' and solved the equations using row operations. No credit was given for this work.
- Q.4 This question was well answered by most candidates.
- Q.5 As reported previously, most candidates knew what had to be done but in many cases the presentation was extremely poor – indeed attempts at solutions using mathematical induction continue to be generally below what can reasonably be expected for candidates working for a qualification in Further Mathematics. It was not uncommon to see a solution beginning:
- Assume true for  $k$ , ie  $\sum \frac{1}{k(k+1)} = \frac{k}{k+1}$
- or even worse,  $\frac{1}{k(k+1)} = \frac{k}{k+1}$
- Candidates should also be encouraged to round off the proof with something along the lines of 'Assuming the result to be true for  $n = k$  implies that the result is true for  $n = k + 1$  and since we have shown it to be true for  $n = 1$ , the general result follows by induction'.
- Q.6 (a) Solutions were generally disappointing with few candidates showing successfully that  $\lambda = 1$  was the only positive root.
- Parts (b) and (c) were generally well done.
- Q.7 Solutions to this question were often disappointing with most candidates replacing  $z$  by  $x + iy$  but then expanding the modulus incorrectly. Some candidates left the  $i$  in the expansion which led to the coordinates of the centre containing  $i$ , a point that did not appear to worry the candidates.

- Q.8 (a)&(b) Were generally well done but few candidates solved (b)(ii) correctly. Many candidates appeared to confuse the starting point  $(x, y)$  and its image  $(x', y')$ . Probably the best way to solve this type of problem is to start with a parametric form of the initial line, here  $(\lambda, 3\lambda + 2)$ , find its image and then eliminate  $\lambda$ .
- Q.9 (a) (i) Solutions were often unconvincing with some candidates going as far as simply writing down the given expression. The later parts of the question were well done in general although some candidates classified the stationary point without reference to the second derivative, not realising that the question was signposted to use that method.

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
FP2	266	75	46.7

#### Grade Ranges

A	52
B	45
C	38
D	31
E	25

*N.B. The marks given above are raw marks and not uniform marks.*

## FP2

### General Comments

The standard of the scripts was generally good with some excellent scripts.

### Comments on Individual Questions

- Q.1 Solutions to this question were generally good although some candidates chose the wrong function in (a).
- Q.2 Solutions to this question were also generally good. The most common errors were algebraic when transforming from an  $x$  integral to a  $u$  integral. It was pleasing to find that most candidates realised why the upper limit could not be changed to  $\pi/3$ .
- Q.3 In converting the given number into polar form, many candidates ended up with the wrong argument by using their calculators to evaluate  $\tan^{-1}\left(\frac{8\sqrt{3}}{-8}\right)$  which gave an angle in the fourth quadrant instead of the second. Candidates should be aware that it is incorrect to state, in general, that  $\arg(x + iy) = \tan^{-1}(y/x)$  since consideration must be given to the signs of  $x$  and  $y$  in order to identify the correct quadrant. This meant that many candidates failed to obtain the correct roots although the follow-through principle enabled them to pick up most of the marks.
- Q.4 Most candidates realised that the best way forward was to combine the  $\sin\theta$  and  $\sin 3\theta$  terms although those candidates who expanded  $\sin 2\theta$  and  $\sin 3\theta$  were sometimes successful. Most candidates successfully converted the values of  $\sin$  and  $\cos$  into the required general solution.
- Q.5 Most candidates found the partial fractions correctly, although some candidates inexplicably let

$$\frac{1}{(x+1)(x+2)(x+3)} \equiv \frac{A}{x+1} + \frac{Bx+C}{(x+2)(x+3)}$$

This is, of course, a correct procedure algebraically but it does not give the partial fractions and it does not help the integration.

- Q.6 Most candidates solved (a) correctly but the subsequent work was sometimes disappointing. Some candidates obtained the required result, ie

$$\frac{a^2}{4x^2} + \frac{b^2}{4y^2} = 1$$

but, not realising that this is a perfectly good equation linking  $x$  and  $y$ , then went on to spend valuable time trying to obtain an explicit expression for  $y$  in terms of  $x$  often making algebraic errors. This further work was of course ignored if incorrect and full credit given for the earlier result.

- Q.7 Many candidates solved (a) correctly although in some cases, the justification for stating that  $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$  was inadequate. Few candidates completed (b) with many stopping at the stage  $\cos\theta = \pi/3$ , presumably believing that was the final answer.

Q.8 Most candidates solved (a) correctly, either by long division or equating coefficients. In (b), it was intended that candidates were to use their expression from (a) to find the derivative of  $f$  but many chose the longer method of differentiating the original expression for  $f(x)$ . Candidates are often unable to detect signposts in examination questions. In (c), the oblique asymptote was often missed. Many candidates seem to be unable to solve problems like that in (d) with many not even knowing how to begin.

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<b>Unit</b>	<b>Entry</b>	<b>Max Mark</b>	<b>Mean Mark</b>
FP3	142	75	42.3

#### Grade Ranges

A	46
B	41
C	36
D	31
E	27

*N.B. The marks given above are raw marks and not uniform marks.*

## FP3

### General Comments

The standard of the scripts was generally good with some excellent scripts. General weaknesses which were spotted include reduction formula where in Q7, there seemed to be a general but incorrect view that these must involve the use of integration by parts. The calculus of hyperbolic functions also caused problems for many candidates.

### Comments on Individual Questions

- Q.1 This question was well answered by most candidates.
- Q.2 Most candidates knew what had to be done but mistakes were made in the successive differentiation.
- Q.3 Many candidates showed that the integral reduced to  $\int \frac{1}{\cosh^2 \theta} d\theta$  but many then replaced the denominator by  $\frac{1}{2}(1 + \cosh 2\theta)$ , which leads nowhere. Few candidates realised that it would be a good idea to introduce  $\operatorname{sech}^2 \theta$  whose integral is known to be  $\tanh \theta$ .
- Q.4 Solutions were often disappointing with algebraic errors often seen. Candidates who used a Cartesian approach were generally more successful than those using parametric coordinates which led to a slightly more difficult integral.
- Q.5 This question was reasonably well answered by many candidates although the sketches in (a) were often poor. In (c), some candidates seemed unsure about whether to look at  $r \cos \theta$  or  $r \sin \theta$ , sometimes looking at both.
- Q.6 Most candidates were unable to derive the recurrence relation in (a). A common method seen was to rewrite  $\tan^n x$  as  $\tan x \cdot \tan^{n-1} x$  and then to attempt to use integration by parts. This led to an even more difficult integral and it did not give the required result. Candidates at this level should have realised from the fact that the relationship linked  $I_n$  and  $I_{n-2}$  that a more useful split was  $\tan^n x = \tan^2 x \cdot \tan^{n-2} x$ , which quickly led to the result using the fact that  $\sec^2 x = 1 + \tan^2 x$ . Most candidates, however, were able to solve (b) correctly.
- Q.7 (a) The majority of candidates stated that, because  $f'(0) = f''(0) = 0$ , there is a stationary point of inflection when  $x = 0$ . This point is, in fact, a maximum as some candidates noted either by stating that  $f$  is an even function or by evaluating  $f(x)$  or  $f'(x)$  on both sides of 0. Candidates should be aware that the equality of these two derivatives to zero is not a sufficient condition for a stationary point of inflection – this can of course be easily demonstrated by considering the behaviour of  $f(x) = x^4$  at the origin.
- (b) Few candidates realised what had to be done in (iii) and some of those who tried to differentiate the hyperbolic function were unable to complete the process.
- (c) Was well done by many of the candidates although the integration of  $\sinh$  and  $\cosh$  sometimes included an unwanted negative sign.



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